Math 116 Sample Exam #3 December, 2018

Instructions: This is a sample exam in preparation for Exam #3, which will be held in class on Monday, Dec. 10, 2018. This sample exam consists of five problems. Do all five, showing your work and explaining your assertions. Give yourself 50 minutes. Each problem is worth 20 points. This sample exam does not have to be handed in. But those who do hand in a completed sample exam at class on Friday, Dec. 7, will get extra credit.

1. Find all points in $\mathbb{R}^2$ where the function $f(x,y) = x^2 - 2x + y^2 - 4y + 7$ has a relative maximum or a relative minimum, indicating which is which.

2. a) Evaluate $\oint_C y\,dx - x\,dy$, where C is the unit circle centered at the origin and oriented counterclockwise.

b) Determine whether the vector field $F = y\,\mathbf{i} - x\,\mathbf{j}$ is the gradient of some function on $\mathbb{R}^2$. If it is, find such a function. If it is not, explain why not. Relate your answer to part (a).

3. a) Let R be a closed rectangular region in the plane, with edges parallel to the axes. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a continuous function with the property that $f(x,y) > 0$ for all points $(x,y) \in R$. Prove that $\iint_R f(x,y)\,dx\,dy > 0$.

b) Let $R \subset \mathbb{R}^2$ be the region in the first quadrant that lies between the circle of radius $\pi/2$ about the origin and the graph of $r = \theta$ (in polar coordinates). Using a double integral, find the area of R.

4. Assume that C is a counterclockwise loop in $\mathbb{R}^2$ that is the boundary of a region $R \subset \mathbb{R}^2$. Let $P = 3x^2y + 6xy$ and $Q = (x+1)^3$, and assume that $\oint_C P\,dx + Q\,dy = 2$. Using Green's Theorem, find the area of R.

5. Let S be the portion of the ellipsoid $x^2/4 + y^2/9 + z^2/25 = 1$ lying above the plane $z = 1$. Let C be the boundary of S, oriented counterclockwise, and with parametrization α. Consider the vector field $F = 2xe^{x^2}yz\,\mathbf{i} + e^{x^2}z\,\mathbf{j} + e^{x^2}y\,\mathbf{k}$.

a) Compute $\operatorname{curl} F$.

b) Evaluate $\oint_C F \cdot d\alpha$. (Hint: Use Stokes' Theorem.)