Read Herstein, Chapter 2, sections 9-11.

1. From Herstein, Chapter 2, do these problems:
   a) Section 2.7, pages 65-66: #15-17. [Hint for #15 and #17: use the first isomorphism theorem.]
   b) Section 2.8, page 70: #5.
   c) Section 2.9, pages 74-75: #4.
   d) Section 2.10, pages 80-81: #1(b), 3(b), 5, 10.
   e) Section 2.11, page 90: #7.

2. Let $G'$ be the commutator subgroup of a group $G$ (see problem 4 on PS #11).
   a) Show that $G/G'$ is well defined and is abelian.
   b) Find $G/G'$ if $G = \mathbb{Z}, \mathbb{Z}/5, S_3, D_4$.

3. Show that $SL_2(\mathbb{R})$ is a normal subgroup of $GL_2(\mathbb{R})$. Which familiar group is isomorphic to $GL_2(\mathbb{R})/SL_2(\mathbb{R})$? [Hint: use the first isomorphism theorem.]

4. a) Let $\phi : G \rightarrow H$ be a homomorphism of groups, and let $N$ be a normal subgroup of $H$. Show that $\phi^{-1}(N)$ is a normal subgroup of $G$.
   b) What does part (a) tell us when $N$ is the trivial subgroup of $H$?

5. Let $G$ be a finite group, let $g \in G$, and let $h = g^r$.
   a) Show that if $o(g)$ is relatively prime to $r$ then $o(g) = o(h)$.
   b) Show that if $r | o(g)$, then $o(h) = o(g)/r$.
   c) Show that in general, $o(h) = o(g)/d$, where $d$ is the greatest common divisor of $r$ and $o(g)$. 