Read Artin, Chapter 3, sections 1-5.

From Artin, do these problems:
- Section 3.1 (p.104): 1, 2.
- Section 3.2 (pp.104-105): 1, 11.
- Section 3.3 (pp.105-106): 2, 10.
- Section 3.4 (pp.106-107): 10.
- Section 3.5 (p.107): 2.


Also do the following problems:

1. a) Let $G = M_2(\mathbb{R})$ under addition. Find a subgroup $H \subset G$ such that for $A, B \in G,$
\[ A \equiv B \pmod{H} \iff \text{trace}(A) = \text{trace}(B). \]

b) Let $G = D_6$ (the group of symmetries of a regular hexagon) and let $v$ be a vertex of the hexagon. Find a subgroup $H \subset G$ such that for $\sigma, \tau \in G,$
\[ \sigma \equiv \tau \pmod{H} \iff \sigma(v) = \tau(v). \]

2. Find all the finite subgroups of the multiplicative group $\mathbb{C}^\times$. Justify your assertion.

3. Define the **commutator subgroup** $G'$ of a group $G$ to be the subgroup of $G$ generated by $\{aba^{-1}b^{-1} \mid a, b \in G\}$.
   a) Find $G'$ if $G = \mathbb{Z}, S_3, D_4$.
   b) Show that $G'$ is a normal subgroup of $G$.
   c) Show that a group $G$ is abelian if and only if $G'$ is the trivial group.
   d) Let $N$ be a normal subgroup of $G$. Show that $G/N$ is abelian if and only if $G' \subset N$.

4. Prove that the functions $e^x, e^{2x}, e^{3x}$ are linearly independent in the vector space $V = \{\text{differentiable functions}\}$. [Hint: If not, then differentiate twice.]

5. Find all real numbers $a$ such that the vectors $(a, 1, 0), (1, a, 1), (0, 1, a)$ are linearly independent in $\mathbb{R}^3$. 