Math 502 Problem Set #11 Due the week of Nov. 20, 2006, in lab or class.

Note: There is no lab this week on Thursday, due to Thanksgiving. Those in the Thursday lab can go to the lab on Tuesday this week, and can submit this homework assignment either in Tuesday lab or in class this week.

Read Artin, Chapter 4, section 8, and Chapter 5, sections 1-8.

From Artin, do these problems:

Section 4.7 (pp.150-151): 2(d), 8 (use part (a) to solve part (b)).
Section 4.8 (pp.151-152): 2, 6, 8.
Section 5.1 (p.188): 3.
Section 5.5 (pp.192-193): 1, 5, 12.
Section 5.6 (pp.193-194): 1, 2.
Section 5.7 (p.194): 1.
Section 5.8 (pp.194-195): 3.
[Optional, for extra credit] Miscellaneous problems for Chap. 4 (pp.152-154): 13.

Also do the following problems:

1. Let G act on a set X, and let $x, x' \in X$.

a) Show that if x, x' are in the same orbit, then their stabilizers are conjugate subgroups of G. Show by example that these stabilizers need not be equal.

b) Show by example that if x, x' are *not* in the same orbit, then their stabilizers need not be conjugate.

2. Describe each of the following compositions of rigid motions of the plane explicitly, either as a translation, rotation, or (glide) reflection:

a) reflection in the x-axis, followed by reflection in the line $y = x$.

b) reflection in the x-axis, followed by reflection in the line $y = 1$.

c) counterclockwise rotation by $\pi/2$ about the origin, followed by translation by $(1, 0)$.

3. Define the *complex upper half plane* $\mathbb{H}$ to be the set of all complex numbers whose imaginary part is positive.

a) Show that $\mathrm{SL}_2(\mathbb{R})$ acts on $\mathbb{H}$ by $A \cdot z = (az + b)/(cz + d)$, where $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

b) Let $N = \{\pm I\}$. Show that N is a normal subgroup of $\mathrm{SL}_2(\mathbb{R})$, and that an element $A \in \mathrm{SL}_2(\mathbb{R})$ leaves *every* element of $\mathbb{H}$ fixed if and only if $A \in N$.

c) Define $\mathrm{PSL}_2(\mathbb{R}) = \mathrm{SL}_2(\mathbb{R})/N$. Show that there is a unique action of $\mathrm{PSL}_2(\mathbb{R})$ on $\mathbb{H}$ such that $\alpha \cdot z = A \cdot z$ for $z \in \mathbb{H}$, if $\alpha \in \mathrm{PSL}_2(\mathbb{R})$ is the image of $A \in \mathrm{SL}_2(\mathbb{R})$ (where the right hand side is given by the action of $\mathrm{SL}_2(\mathbb{R})$).

d) Is the action of $\mathrm{SL}_2(\mathbb{R})$ on $\mathbb{H}$ faithful? transitive? What about the action of $\mathrm{PSL}_2(\mathbb{R})$?

[*Remark*: If $\mathbb{H}$ is given the geometric structure for which $\mathrm{PSL}_2(\mathbb{R})$ is the group of symmetries, then $\mathbb{H}$ becomes a non-Euclidean space, called the *Siegel upper half plane*.]