Math 502 Problem Set #13 Due the week of Dec. 4, 2006, in Tues. lab or in class.

Reminders: There will be an exam in class on Wed., Dec. 6, focusing on the material covered since the first exam. The lab on Tues., Dec. 5 will also serve as a review session, and will be open to everyone in the class.

Read Artin, Chapter 6, sections 7-8, and Chapter 7 (all).

From Artin, do these problems:

Section 6.6 (pp.232-233): 5.
Section 6.7 (pp.233-234): 1.
Section 6.8 (p.234): 6, 7. [Note: There is a misprint in #6, which should read, "Assume that N and G/N are both cyclic groups." Also, the two elements generating G are allowed to be equal.]
Section 7.1 (pp.262-263): 3.
Section 7.2 (pp.263-264): 3(b).
Section 7.3 (pp.264-265): 2.
Section 7.4 (pp.265-266): 5.
Section 7.5 (pp.266-267): 4, 5(a).

Also do the following problems:

1. a) Show that if $T : V \to V$ is a linear transformation, and $v \in V$ is an eigenvector for T with eigenvalue c, then v is also an eigenvector for T^k, with eigenvalue c^k.

b) Use this to find the eigenvalues and corresponding eigenvectors of the matrix A^{253}, where $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & -2 \\ 0 & 0 & -1 \end{pmatrix}$.

2. In $\mathbb{R}^4$, let V be the span of $(1,0,1,0)$ and $(1,1,3,1)$.

a) Find an orthonormal basis of V.
b) Express $(1,2,3,4) = v_1 + v_2$ explicitly, for some $v_1 \in V$ and $v_2 \in V^\perp$.
c) Find the point of V that is closest to $(1,2,3,4)$.
d) Find an orthonormal basis of $V^\perp$.

3. Over $\mathbb{R}$, which of the following matrices have an orthonormal basis of eigenvectors? What about over $\mathbb{C}$? [Note: You are not required to find the basis explicitly.]

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix} \qquad B = \begin{pmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$$

Also, which of these matrices are symmetric? orthogonal? Hermitian? unitary?