Read Artin, Chapter 7, sections 4, 5, 9 (optional: 8, 10), and Chapter 8 (all).

1. From Artin, do these problems:
   a) From Chapter 7 (pages 221-228): 5.3, 7.4(a).
   b) From Chapter 8 (pages 254-260): 3.3, 4.7, 5.2, 5.6, 6.11, 6.12, 6.13(a).

2. a) Show that if \( T : V \to V \) is a linear transformation, and \( v \in V \) is an eigenvector for \( T \) with eigenvalue \( c \), then \( v \) is also an eigenvector for \( T^k \), with eigenvalue \( c^k \).
   b) Use this to find the eigenvalues and corresponding eigenvectors of the matrix \( A^{253} \), where \( A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & -2 \\ 0 & 0 & -1 \end{pmatrix} \).

3. In \( \mathbb{R}^4 \), let \( V \) be the span of \((1, 0, 1, 0)\) and \((1, 1, 3, 1)\).
   a) Find an orthonormal basis of \( V \).
   b) Express \((1, 2, 3, 4) = v_1 + v_2 \) explicitly, for some \( v_1 \in V \) and \( v_2 \in V^\perp \).
   c) Find the point of \( V \) that is closest to \((1, 2, 3, 4)\).
   d) Find an orthonormal basis of \( V^\perp \).

4. Over \( \mathbb{R} \), which of the following matrices have an orthonormal basis of eigenvectors? What about over \( \mathbb{C} \)? [Note: You are not required to find the basis explicitly.]

\[
A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix} \quad B = \begin{pmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}
\]

Also, which of these matrices are symmetric? orthogonal? Hermitian? unitary?