Math 502 Problem Set #3 Due week of Sept. 18, 2017, in lab.

Read Artin, Chapter 2, sections 3 and 4.

1. From Artin, Chapter 2, do these problems (pages 69-77):
3.1, 3.2, 4.2, 4.3, 4.5, 4.6.

2. Determine whether the following maps between groups are homomorphisms and whether they are isomorphisms. For each homomorphism, find the kernel and the image. (Here $n \geq 3$.)

a) $C_n \to D_n$, taking the j-th power of the generator c of C_n to the rotation ρ^j, where ρ is the counterclockwise rotation by an angle of $2\pi/n$.

b) $D_n \to C_n$, taking ρ^j to c^j and taking each flip to the identity.

c) $D_n \to S_n$, taking each symmetry of the regular n-gon to the permutation of the vertices that the symmetry determines.

d) $D_n \to C_2$, taking each rotation to the identity, and taking each flip to the generator c of C_2.

3. Use the well ordering principle for the positive integers to prove that 1 is the least positive integer. (Hint: If m is the least positive integer, is $m^2 < m$?)

4. Show that the following three assertions are equivalent (i.e., each one implies the others):

a) The set of positive integers is well ordered.

b) The principle of induction: If S is a subset of the positive integers that contains 1, and if $n + 1 \in S$ whenever $n \in S$, then S is the set of positive integers.

c) The principle of complete induction: If S is a subset of the positive integers, and if S has the property that $n \in S$ whenever all smaller positive integers are in S, then S is the set of positive integers.

(Hint: For (a) $\Rightarrow$ (b) and for (c) $\Rightarrow$ (a), take the complementary set and use contradiction. For (b) $\Rightarrow$ (c), use problem 3.)

5. a) Find the greatest common divisor of 943 and 667 in each of the following two ways, in each case showing your work:

i) By factoring each of the integers into a product of prime numbers.

ii) By using the Euclidean algorithm, as described in Artin (bottom of page 44 to top of page 45).

b) Explain the advantages and disadvantages of each of these two methods for finding the greatest common divisor of two integers, depending on whether the given integers are very large (e.g., around 10^{100}) or relatively small.