Math 502 Problem Set #6 Due week of Oct. 9, 2017, in lab.

Review Artin, Chapter 2, and read Section 3.2.

1. From Artin, Chapter 2, do these problems (pages 69-77):
9.4, 9.7, 10.4, 11.1, 11.5, 11.6. (Note: In 10.4, $G' = \mathcal{G}$ and $H' = \mathcal{H}$.) (Hint for 11.6: Use one of the propositions in that section.)

2. Find all integers n such that there is an element of D_7 having order n. [Hint: If a group of order m has an element of order m, what can you say about the group?]

3. Define the *commutator subgroup* G' of a group G to be the subgroup of G generated by $\{aba^{-1}b^{-1} \mid a, b \in G\}$.

a) Find G' if $G = \mathbb{Z}, S_3, D_4$.

b) Show that G' is a normal subgroup of G.

c) Show that a group G is abelian if and only if G' is the trivial group.

d) Let N be a normal subgroup of G. Show that G/N is abelian if and only if $G' \subseteq N$.

4. a) Find all the finite subgroups of the multiplicative group $\mathbb{C}^\times$ of the field $\mathbb{C}$. Justify your assertion.

b) Do the same for the multiplicative group of the field $\mathbb{Z}/7\mathbb{Z}$.

5. a) Let $\mathbb{R}^\times$ be the multiplicative group of $\mathbb{R}$. Find a subgroup $H \subset \mathbb{R}^\times$ such that for $x, y \in \mathbb{R}^\times$,

$$x \equiv y \pmod H \Leftrightarrow x/|x| = y/|y|.$$

b) Let $G = M_2(\mathbb{R})$ under addition. Find a subgroup $H \subset G$ such that for $A, B \in G$,

$$A \equiv B \pmod H \Leftrightarrow \text{trace}(A) = \text{trace}(B).$$

c) Let $G = D_6$ (the group of symmetries of a regular hexagon) and let v be a vertex of the hexagon. Find a subgroup $H \subset G$ such that for $\sigma, \tau \in G$,

$$\sigma \equiv \tau \pmod H \Leftrightarrow \sigma(v) = \tau(v).$$