Reminder: There will be an exam in class on Wednesday, Oct. 25. This problem set is a sample exam. Those who complete this and submit it in class on Monday, Oct. 23, will be given extra credit.

1. Let H be the set of $n \times n$ complex matrices M such that $|\det M| = 1$. Show that H is a subgroup of $\mathrm{GL}_n(\mathbb{C})$. Is H normal in $\mathrm{GL}_n(\mathbb{C})$?

2. Find two non-isomorphic groups of order 18. Explain.

3. Find all integers n such that the dihedral group D_{11} has an element of order n.

4. Find all real numbers c such that the subset $V_c = \{(x, y, z) \in \mathbb{R}^3 \,|\, x + y + z = c\}$ is a subspace of $\mathbb{R}^3$. For each such c, find a basis of V_c and find the dimension of V_c.

5. Consider the action of S_3 on its subgroup C_3 given by conjugation. Find the orbits and the stabilizers.

6. Find all integers n such that the equation $21x + 36y = n$ has a solution in integers x, y. Justify your assertion.

7. Determine if there is a homomorphism from S_3 to some group G whose kernel has order 2.

8. Is there a field of six elements? a field of seven elements? Justify your assertions.