Read Artin, Chapter 12, sections 5-7.

Part A:
From Artin, do these problems (given at the end of Chapter 12):
12.5: 5; 12.6: 1, 2, 4; 12.7: 1, 2 (also find the rational canonical form), 6.

Part B:
1. Determine which of the following rings are Noetherian:
   \( \mathbb{Z}[i, x, y]/(x^2 + iy^3); \ \mathbb{R}[[x]][y]; \ \mathbb{Z}[\sqrt{2}, \sqrt{2}, \sqrt{2}, \ldots]; \ \mathbb{Z}[1/2]; \ \mathbb{Z}[1/2, 1/3, 1/4, \ldots]. \)

2. a) In the situation of Problem Set #7, problem 3, suppose that \( R \) is a field. Show that if \( \phi \) is surjective then so is \( \phi^* \). Also show that if \( \psi \) is injective then \( \psi^* \) is surjective.
   b) Show by example that the conclusion of part (a) does not necessarily hold for modules over an arbitrary commutative ring \( R \). [Hint: Take \( R = \mathbb{Z} \), let one of the modules be \( \mathbb{Z}/2 \), and let one of the maps be multiplication by 2.]

3. Let \( G \) be a group. For any positive integer \( n \), let \( G_n \) consist of the elements of \( G \) whose order divides \( n \). Let \( G_{\text{tor}} \) be the union of all the sets \( G_n \). (So the elements of \( G_{\text{tor}} \) are the elements of \( G \) having finite order; these are also called “torsion elements.”)
   a) Show that if \( G \) is abelian, then \( G_n \) and \( G_{\text{tor}} \) are subgroups of \( G \).
   b) Show by example that if \( G \) is non-abelian then these subsets need not be subgroups of \( G \). [Hint: For \( G_{\text{tor}} \), consider the infinite dihedral group.]

4. Let \( G = \prod_{i=1}^{s} \mathbb{Z}/d_i \). In the notation of problem 3, show that \( G_n \) is isomorphic to \( \prod_{i=1}^{s} \mathbb{Z}/(n, d_i) \), where \( (n, d_i) \) is the greatest common divisor of \( n \) and \( d_i \).

Part C:
From Artin, do these problems (at the end of Chapter 12):
Section 12.7: 18 (do this using rational canonical form; first try the case of one block);
Miscellaneous problems: 7, 11 [Hint: Pick \( g \in A \) and make the substitution \( a = bg \), where \( b \) is a new variable.]

Also do the following problem:

Suppose that \( \prod_{i=1}^{s} \mathbb{Z}/d_i \) is isomorphic to \( \prod_{i=1}^{s'} \mathbb{Z}/d'_i \), where \( d_1 | d_2 | \cdots | d_s \) and \( d'_1 | d'_2 | \cdots | d'_{s'} \).
Prove \( s = s' \) and \( d_i = d'_i \) for all \( i \). [Hint: Use problem 4 above for suitable choices of \( n \).]