Read Artin, Chapter 13, sections 1-3.

**Part A:**
From Artin, do these problems (given at the end of Chapter 12): Miscellaneous problems 8, 9.
From Artin, do these problems (given at the end of Chapter 13): Section 13.1: 1, 4; 13.2: 1; 13.3: 1, 2.

**Part B:**
1. a) If $R$ is a commutative ring, find an isomorphism $R \otimes \mathbb{Z} [\sqrt{2}] \cong R[x]/(x^2 - 2)$.
   b) Determine whether $\mathbb{Z}[\sqrt{3}] \otimes \mathbb{Z} [\sqrt{2}]$ and $\mathbb{Z}[\sqrt{2}] \otimes \mathbb{Z} [\sqrt{2}]$ are integral domains.
   c) Simplify each of the following $\mathbb{Z}$-modules (up to isomorphism): $\mathbb{Z}/10 \otimes \mathbb{Z} [\sqrt{2}]$, $\mathbb{Z}/10 \otimes \mathbb{Z} [\sqrt{2}] / 6$, $\mathbb{Z}/10 \otimes \mathbb{Q}$, $\mathbb{Q} \otimes \mathbb{Z}$, $\mathbb{Q} \otimes \mathbb{Z} / 3$.

2. Let $R$ be a commutative ring, let $\phi : N_1 \rightarrow N_2$ be a homomorphism of $R$-modules, and let $M$ be an $R$-module.
   a) Show that there is an induced homomorphism of $R$-modules $\phi_* : M \otimes N_1 \rightarrow M \otimes N_2$ defined by $\phi_* (m \otimes n) = m \otimes \phi(n)$.
   b) Show that if $\phi$ is surjective then so is $\phi_*$.
   c) Show that if $M$ is a free $R$-module (e.g. if $R$ is a field), then if $\phi$ is injective then so is $\phi_*$. But show by example that if $M$ is arbitrary, then it is possible for $\phi$ to be injective and $\phi_*$ not to be injective. [Compare PS #9, problem B2.]

3. For each of the following field extensions $F$ of $\mathbb{Q}$, find the degree of $F$ over $\mathbb{Q}$ and find the group $\text{Aut}(F)$ of automorphisms of $F$.
   $\mathbb{Q}$, $\mathbb{Q}[\sqrt{5}]$, $\mathbb{Q}[\zeta_5]$ (where $\zeta_5$ is a primitive fifth root of unity), $\mathbb{Q}[\sqrt{2}]$, $\mathbb{Q}[\sqrt[4]{2}]$

**Part C:**
From Artin, do these problems (at the end of Chapter 13):
Section 13.1: 2, 3; 13.2: 3.

Also do the following problem:
Let $K = \mathbb{Q}[\sqrt{2}]$ and $L = \mathbb{Q}[\sqrt{2} + \sqrt{2}]$.
   a) Find the multiplicative inverse of $\sqrt{2} + \sqrt{2}$ in $L$ (as a polynomial in $\sqrt{2} + \sqrt{2}$).
   b) Show $K \subset L$. What is $[K : \mathbb{Q}]$? $[L : K]$? $[L : \mathbb{Q}]$?
   c) Let $\phi$ be an automorphism of $L$. What can you say about the restriction $\phi|_{\mathbb{Q}}$?
   d) Let $\phi$ be an automorphism of $L$. What can you say about the restriction $\phi|_{K}$?
   e) Find an element of order 4 in $\text{Aut}(L)$. What is the group $\text{Aut}(L)$ abstractly?