Math 503 Problem Set #2 Due the week of Jan. 22, 2018, in lab.

Read Artin, Chapter 11, sections 2-3.

1. From Artin, Chapter 11, do these problems (pages 354-358):
1.3, 1.8, 2.2, 3.8, 3.10.

2. a) In Part A of Problem Set #1, determine which of the items are rings with zero-divisors, and which of the items are fields.

b) In Part B of Problem Set #1, find the kernel of each item that is a ring homomorphism. For each non-zero kernel, find an explicit non-zero element of the kernel.

3. a) Find a ring homomorphism $\mathbb{Q}[x] \to \mathbb{R}$ whose kernel is the principal ideal $(x^3 - 2)$.

b) Show that there is a *unique* ring homomorphism $\mathbb{Q} \to \mathbb{R}$.

c) Show that there is *no* ring homomorphism $\mathbb{R} \to \mathbb{Q}$.

4. Let $\mathbb{H} = \{\text{quaternions } a + bi + cj + dk \mid a, b, c, d \in \mathbb{R}\}$.

a) For $\alpha = a + bi + cj + dk \in \mathbb{H}$, define its *conjugate* $\bar{\alpha} = a - bi - cj - dk$, and define its *absolute value* to be the non-negative real number $|\alpha| = \sqrt{a^2 + b^2 + c^2 + d^2}$. Show that $|\alpha|^2 = \alpha\bar{\alpha}$ and that $\overline{\alpha\beta} = \bar{\beta}\bar{\alpha}$. Conclude that $|\alpha\beta| = |\alpha||\beta|$. Also, find all α such that $|\alpha| = 0$.

b) Does $\mathbb{H}$ have any zero-divisors? (Hint: Use part (a).)

c) Is $\mathbb{H}$ a division ring (i.e., does every non-zero element have an inverse)? Is $\mathbb{H}$ a field?

5. Let F be a field, and let $R = F[[t]]$ = the ring of formal power series in t over F. Show that the set of non-units in R is a principal ideal (f) of R, for some $f \in R$; and find f explicitly. Show also that $R[1/f]$ is a field, and describe what the elements of this field look like.