Math 503 Problem Set #4 Due the week of Feb. 5, 2018, in lab.

Read Artin, Chapter 11, section 9.

1. From Artin, Chapter 11, do these problems (pages 354-358): 6.2, 6.5, 7.5, 8.2, 8.4, 9.1. (In problem 8.4, the *upper half plane* is $\{a + bi \mid a, b \in \mathbb{R}, b \geq 0\}$.)

2. In Problem Set 3, problem 4, does the assertion in part (a) still hold if R is a non-commutative ring with no zero-divisors? Either prove that it must still hold, or else give a counterexample and say where your argument for part (a) of that problem breaks down. [Hint: Consider $R = \mathbb{H}$ and the polynomial $x^2 + 1$. If $f(x) = (x - a)(x - b) \in R[x]$ and $c \in R$, must $f(c) = (c - a)(c - b)$?]

3. Let $f(x, y) = x^2 + y^2 + 1$.

a) Is the principal ideal $(f) \subset \mathbb{C}[x, y]$ maximal? If not, find a maximal ideal containing it.

b) Do the same with $\mathbb{C}$ replaced by $\mathbb{R}$.

4. Find the maximal ideals of $\mathbb{Z}[1/2]$. Do the same for $\mathbb{C}[x, x^{-1}]$.

5. Let S be a ring and let $R \subset S$ be a subring. If I is an ideal of S, define its *contraction* I^c to R to be $I \cap R$.

a) Show that I^c is an ideal of R.

b) If I is a maximal ideal of S, must I^c be a maximal ideal of R? Do this in the following two cases:

i) $R = \mathbb{C}[x]$, $S = \mathbb{C}[x, y]$.

ii) $R = \mathbb{C}[x]$, $S = \mathbb{C}(x)$.