Math 503 Problem Set #6 Due the week of Feb. 19, 2018, in lab.

Reminder: There will be an exam in class on Wednesday, Feb. 21. This problem set is a sample exam. Those who complete this and submit it in class on Monday, Feb. 19, will be given extra credit. As on the actual exam, you should do all the problems, showing your work and explaining your assertions.

Read/review Artin, Chapters 11 and 12 (all).

Part I:

1. Find a maximal ideal of $\mathbb{C}[x, y]$ that contains the element $x^3 + y^3 - 1$.

2. Show that $(x^2 + 1, y^2 + 1)$ is not a maximal ideal of $\mathbb{R}[x, y]$.

3. Show that the Gaussian integers $5 + i$ and $4 + 3i$ are relatively prime.

4. Determine whether the polynomial $x^5 + 6x^4 - 15x^2 + 3$ is irreducible in $\mathbb{Z}[x]$. Justify your assertion.

Part II:

5. Determine whether $(x^2 + 2)$ is a maximal ideal of $\mathbb{Q}[x]$, and whether $\mathbb{Q}[\sqrt{-2}]$ is a field.

6. Let I be the set of matrices in $M_2(\mathbb{R})$ whose second row is $(0\ 0)$. Show that I is a right ideal of $M_2(\mathbb{R})$ but not a two-sided ideal.

7. Show that every non-zero element of $\mathbb{C}[x]/(x^2)$ is either a zero-divisor or a unit.

8. a) Determine whether there exist integers $a, b \in \mathbb{Z}$ such that $a^2+b^2 = 8009$, and whether there is a $\sqrt{-1} \in (\mathbb{F}_{8009})^{\times}$.

b) Do the same with 8009 replaced by 8011.

[Note: 8009 and 8011 are prime numbers. Also, you are not asked to *find* a and b, or a $\sqrt{-1}$, just to determine if they exist.]