Math 503 Problem Set #12 Due the week of April 9, 2018, in lab.

In Artin, Chapter 15, read sections 5-8 (optional: 9-10).

1. From Artin, Chapter 15, do these problems (pages 472-476): 5.3, 5.4, 6.3, 7.3, 7.7. [Note: In 6.3, $n > 2$. As a hint, use that $\mathbb{Q}(\zeta_p) \subseteq \mathbb{Q}(\zeta_n)$ if $p|n$. Also use problem 3 below.]

2. a) Given a point P in the plane and a circle C with center P, show that for every positive integer n one can construct (with straightedge and compass) a circle whose circumference is n times that of C, and a circle whose area is n times that of C.

b) What happens in instead one is given a point P in 3-space and a sphere S with center P, and one wants to find spheres whose surface area or volume is n times that of S? [Note: For a sphere of radius r, the surface area is $4\pi r^2$ and the volume is $\frac{4}{3}\pi r^3$.]

3. a) Show that if K is a field whose characteristic is not 2, and if $L = K[\sqrt{a}]$ and $L' = K[\sqrt{b}]$ are two field extensions of K of degree 2, then L and L' are isomorphic as field extensions of K if and only if there exists $c \in K$ such that $b = ac^2$.

b) Show that this fails in characteristic 2. [Hint: Take $K = \mathbb{F}_2(t)$.]

4. Let p be a prime number, and let r, s be positive integers. Show that r divides s if and only if $p^r - 1$ divides $p^s - 1$.

5. a) Find the degree of $\alpha = \sqrt{3} + \sqrt{5}$ over $\mathbb{Q}$, and also find its minimal polynomial.

b) Determine whether $\mathbb{Q}(\alpha) = \mathbb{Q}(\sqrt{3}, \sqrt{5})$.

c) Repeat part (a) for $\beta = \sqrt[4]{2}$.

d) For each of the fields $\mathbb{Q}(\sqrt{3}, \sqrt{5})$ and $\mathbb{Q}(\beta)$, compare the degree of the field over $\mathbb{Q}$ with the order of the group of automorphisms of the field.