1. Let \( n > 2 \). Show that if the dihedral group \( D_n \) of order \( 2n \) is isomorphic to a semi-direct product \( C_r \rtimes C_s \), then \( r = n \) and \( s = 2 \).

2. Show that \( A_4 \) is isomorphic to a semi-direct product \( C_2^2 \rtimes C_3 \).

3. Which of the following groups are isomorphic: \( C_2 \wr C_2 \), \( Q_8 \), \( D_4 \), \( C_2^3 \)?

4. Find all groups of order 66, up to isomorphism. Which are simple? solvable? nilpotent? abelian? cyclic? Which are split extensions (of a non-trivial quotient by a non-trivial subgroup)?

5. a) Show directly that every group of order 56 is solvable. [Hint: How many elements have order 7?]
   
   b) Consider the finite groups whose order is 56 and whose exponent is 14. For each such group, let \( N_p \) be the number of Sylow \( p \)-subgroups, for \( p = 2, 7 \).
   
   (i) Do there exist such groups with \( N_2 = N_7 = 1 \)?
   
   (ii) Do there exist such groups with \( N_7 = 1 \) and \( N_2 > 1 \)?
   
   (iii) Do there exist such groups with \( N_2 = 1 \) and \( N_7 > 1 \)?
   
   (iv) Do there exist such groups with \( N_2 > 1 \) and \( N_7 > 1 \)?

6. Find two extensions \( G \) of a fixed finite group \( B \) by a fixed finite abelian group \( A \) such that the two groups \( B \) are isomorphic as groups, but such that the two extensions \( 1 \to A \to G \to B \to 1 \) are not isomorphic as extensions of \( B \) by \( A \). [Hint: Try \( A = C_3^2 \) and \( B = C_2 \).]

7. Show that there is a unique action of \( C_2 \) on \( C_2 \). With respect to that action, directly compute the groups \( C^2(C_2, C_2) \), \( Z^2(C_2, C_2) \), \( B^2(C_2, C_2) \), \( H^2(C_2, C_2) \). In the case of \( H^2 \), interpret each element in terms of an extension of \( C_2 \) by \( C_2 \).

8. Let \( 0 \to A \xrightarrow{i} G \xrightarrow{\pi} B \to 1 \) be a short exact sequence of finite groups, with \( A \) abelian (written additively). For each \( b \in B \) pick some \( g_b \in G \) such that \( \pi(g_b) = b \). Define an action \( \alpha \) of \( B \) on \( A \) by \( b \cdot a = g_b \alpha g_b^{-1} \). For \( b_1, b_2 \in B \), define \( f(b_1, b_2) \in A \) by \( g_{b_1} g_{b_2} = f(b_1, b_2) g_{b_1 b_2} \).
   
   a) Show that \( f \in Z^2_\alpha(B, A) \); i.e. that \( f(b_1, b_2) + f(b_1 b_2, b_3) = b_1 \cdot f(b_2, b_3) + f(b_1, b_2 b_3) \).
   
   [Hint: Evaluate \( g_{b_1} g_{b_2} g_{b_3} \) in two ways.]

   b) Show that \( (a_1 g_{b_1}) (a_2 g_{b_2}) = (a_1 + b_1 \cdot a_2 + f(b_1, b_2)) g_{b_1 b_2} \in G \) for \( a_1, a_2 \in A \) and \( b_1, b_2 \in B \), giving the multiplication law in \( G \).

   c) Suppose that for each \( b \in B \) we have another choice \( g'_b \in G \) of an element in \( G \) with \( \pi(g'_b) = b \), and let \( f' \) be the analogous element of \( Z^2_\alpha(B, A) \). For each \( b \in B \) define \( e(b) \in A \) by \( g'_b = e(b) g_b \). Show that \( f'(b_1, b_2) - f(b_1, b_2) = e(b_1) + b_1 \cdot e(b_2) - e(b_1 b_2) \); i.e. \( f, f' \) differ by an element of \( B^2_\alpha(B, A) \). [Hint: Evaluate \( g'_{b_1} g'_{b_2} \) in two ways.]