For each of the following, either give an example, or else prove that none exists.

1. A non-abelian group of order 55.
2. A non-abelian group of order 121.
3. A simple group of order 256.
4. A finite group that is solvable but not nilpotent.
5. A non-split short exact sequence of finite groups.
6. A finite abelian group whose automorphism group is non-abelian.
7. A non-trivial proper left ideal in $M_3(F_5)$ (i.e. $3 \times 3$ matrices over $F_5 = \mathbb{Z}/5\mathbb{Z}$).
9. A maximal ideal in $\mathbb{R}[x, y]/(xy - 2)$.
10. A local ring that is an integral domain but not a field.
11. A commutative ring whose nilradical is unequal to its Jacobson radical.
12. An ideal $I$ in an integral domain $R$ such that $I$ is radical but not prime.
13. A non-zero proper ideal of $\mathbb{Q}[[x]]$ that is not maximal.
14. A matrix in $M_2(\mathbb{Q})$ that is triangularizable over $\mathbb{C}$ but not over $\mathbb{R}$.
15. A non-zero alternating 3-form on $\mathbb{R}^2$.
16. A $4 \times 4$ real orthogonal matrix of rank 3.
17. A set of eight linearly independent vectors in $\mathbb{R}^3 \otimes \mathbb{R}^4$.
18. An invertible linear transformation $T : \mathbb{R}^5 \to \mathbb{R}^5$ such that $Tv \perp v$ for all $v \in \mathbb{R}^5$.
19. A quadratic form $q$ over a field $K$ such that $q$ is both isotropic and regular.
20. Two unequal symmetric bilinear forms $B, B'$ on a $\mathbb{Q}$-vector space $V$ such that $B(v, v) = B'(v, v)$ for all $v \in V$. 