

Part I:

Read Fulton, Chapter 1, sections 8-10, and Chapter 2, sections 1-4.

Do problems:

- 1.40 in Section 1.7;
- 2.11-2.13 in Section 2.2;
- 2.16 in Section 2.3;
- 2.18, 2.21 in Section 2.4.

Part II:

Read Hartshorne, Chapter I, section 4.

1. In Chapter I, do problems:

- 2.14, 2.15, 3.4, 3.6, 3.14-3.16, 4.4, 4.5.

2. (a) Show that the complex affine line is not birationally isomorphic to the curve $X \subset \mathbb{A}_{\mathbb{C}}^2$ given by $x^3 + y^3 = 1$. Do this in steps, by assuming instead that there *is* an isomorphism $\Phi : K(\mathbb{A}^2) \rightarrow K(X)$ of function fields, and then proceeding as follows:

(i) There are relatively prime non-zero polynomials $p, q, r \in \mathbf{C}[t]$ such that $p^3 + q^3 - r^3 = 0$ identically.

(ii) For these p, q, r , there is the matrix identity

$$\begin{pmatrix} p & q & r \\ p' & q' & r' \end{pmatrix} \begin{pmatrix} p^2 \\ q^2 \\ -r^2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

where p', q', r' are the derivatives of p, q, r .

(iii) Hence $qr' - rq'$, $rp' - pr'$, $pq' - qp'$ are respectively divisible by p^2, q^2, r^2 .

(iv) By examining the degrees of p, q, r , derive a contradiction.

(b) What if the base field is some other algebraically closed field k , instead of \mathbb{C} ? Does the conclusion of part (a) remain true for *all* such k ?

3. Let $X = \mathbb{A}^2$, let $P \in X$ be the origin, and let $\pi : \tilde{X} \rightarrow X$ be the blow-up of X at P . Let ϕ be the rational map from X to \mathbb{P}^1 that sends (x, y) to $(x : y)$. Show explicitly that ϕ lifts to a morphism $\tilde{\phi}$ defined on \tilde{X} ; i.e. there exists a morphism $\tilde{\phi} : \tilde{X} \rightarrow \mathbb{P}^1$ that defines the same rational map as $\phi \circ \pi$.