Part I:
Read Fulton, Chapter 6, sections 4-6.
Do problems:
5.38 on pp.127;
6.27, 6.29, 6.30 on pp.147-148;

Part II:
Read Hartshorne, Chapter II, sections 6-8. (Section 9 is optional.)
1. In Chapter II, do problems 6.6, 7.1.
(Additional study problems: 6.2, 6.4, 6.10, 7.6, 7.7, 7.13, 8.2, 8.3.)
2. a) Show that the quartic (degree 4) curves in $\mathbb{P}^2$ form a complete linear system, and find its dimension $d$. (Here “curve” means the scheme defined by the ideal of a homogeneous polynomial, and degenerate curves are permitted.)
   b) Let $P$ be a closed point of $\mathbb{P}^2$, and consider the curves in the linear system in (a) that pass through $P$. Show that they form a linear system, and find its dimension. Is this a complete linear system?
   c) Redo part (b) with $P$ replaced by two distinct points $P, Q$ in $\mathbb{P}^2$ (i.e. curves passing through both points).
   d) Does the obvious pattern of dimensions, suggested by your answers to parts (a)-(c), continue indefinitely if more and more points are chosen?
3. Let $k$ be an algebraically closed field, and let $X$ be a smooth connected projective curve that is not isomorphic to $\mathbb{P}_k^1$. Let $K$ be the function field of $X$. Let $f \in K - k$.
   a) Show that $f$ defines a non-constant rational map from $X$ to $\mathbb{P}^1$, and that this extends to a morphism $X \to \mathbb{P}^1$.
   b) Deduce that the divisor $(f)_\infty$ has degree $> 1$. [Hint: What is the degree of the morphism in (a)?]
   c) Deduce that if $P$ is a closed point of $X$, then there is no rational function on $X$ having a pole of order 1 at $P$ and having no other poles.
   d) Conclude that if $P, Q \in X$ are distinct closed points, then viewed as divisors, $P$ and $Q$ are not linearly equivalent.
   e) Evaluate the dimensions of the $k$-vector spaces $\Gamma(X, O)$ and $\Gamma(X, O(P))$, where $P$ is a closed point of $X$.
   f) Do your answers to parts (a)-(e) change if we instead take $X = \mathbb{P}^1$?
4. a) Show directly, by considering differential forms, that $\Omega^1_X \approx O(-2)$ if $X = \mathbb{P}^1$.
   b) Show directly, by considering differential forms, that $\Omega^1_X \approx O$, if $X \subset \mathbb{P}^2$ is the cubic curve given by $y^2z = x^3 - xz^2$.
   c) Verify Riemann-Roch directly for $X = \mathbb{P}^1$. That is, show that for any divisor $D$,
      $$\dim \Gamma(X, O(D)) - \dim \Gamma(X, \Omega^1_X \otimes O(-D)) = \deg D + 1 - g,$$
   where $g = \text{genus}(\mathbb{P}^1) = 0$. [Hint: $D \sim nP$ for some $n \in \mathbb{Z}$ and any point $P$ on $\mathbb{P}^1$.]