1. In Hartshorne, Chapter III, do problems 4.5 (for a Noetherian separated scheme), 9.4, 10.2, 11.2, 11.3.
   Optional: problems 9.11, 10.1, 10.3, 10.5, 10.6, 11.4, 12.2.

2. For each of the following morphisms \( \phi \), determine whether \( \phi \) is of finite type, finite, quasi-finite, proper, surjective, projective, flat, and étale. (Below, \( k \) is an arbitrary field.)
   (i) \( \phi \) is the morphism corresponding to the inclusion of rings \( k[x] \hookrightarrow k[x, y]/(y^2 - x^2) \).
   (ii) \( \phi \) is the morphism corresponding to the inclusion of rings \( k[x] \hookrightarrow k[[x]] \).
   (iii) \( \phi \) is the morphism corresponding to the inclusion of rings \( k[x] \hookrightarrow k[x, y]/(y^3 - x) \).
   (iv) \( \phi \) is the morphism corresponding to the inclusion of rings \( k[x] \hookrightarrow k[x, x^{-1}, y]/(y^3 - x) \).
   (v) \( \phi \) is the morphism corresponding to the inclusion of rings \( k[x] \hookrightarrow k[y]/(xy) \).
   (vi) \( \phi \) is the morphism corresponding to the inclusion of rings \( k[x, y] \hookrightarrow k[x, y, z]/(z^2 - xy) \).
   (vii) \( \phi \) is the morphism corresponding to the inclusion of rings \( k[x, y, z]/(z^2 - xy) \hookrightarrow k[u, v] \) given by \( x \mapsto u^2, y \mapsto v^2, z \mapsto uv \).
   (viii) \( \phi : E \rightarrow E \) is multiplication by 3 on an elliptic curve over \( k \).

3. Let \( k \) be a field. Which of the following ideals \( I \subset k[[x, y]] \) is maximal? prime? the unit ideal? In each case, describe geometrically the locus of \( I \) in \( \text{Spec} \ k[[x, y]] \).
   \( I = (x), (x, y), (xy), (1 - xy), (x - y), (y^2 - x^2), (y^2 - x^3), (y^2 - x^2 - x^3) \).

4. Let \( f : Y \rightarrow X \) be a finite étale cover of smooth connected schemes, say of degree \( n \). Show that there is a finite étale cover \( Z \rightarrow X \) such that the pullback \( Y \times_X Z \rightarrow Z \) is a trivial cover, consisting of \( n \) disjoint copies of \( Z \). Explain why this says that finite étale covers are covering spaces in the étale topology. Contrast this with what happens in the Zariski topology.

5. Suppose that \( f : Y \rightarrow X \) is a birational morphism of smooth projective varieties, and let \( H \subset Y \) be a hypersurface whose image has dimension less than that of \( H \). Prove that \( H \) is not linearly equivalent to any effective divisor on \( Y \) that meets \( H \) properly. [Hint: Otherwise, consider the corresponding rational function on \( Y \), and view it as a rational function on \( X \). What is its divisor there?]