

13.3

$$5. \quad r(t) = \cos^3 t \hat{j} + \sin^3 t \hat{k}$$

$$\begin{aligned} r'(t) &= 3\cos^2 t (-\sin t) \hat{j} + 3\sin^2 t \cos t \hat{k} \\ &= 3\cos t \sin t (-\cos t \hat{j} + \sin t \hat{k}) \end{aligned}$$

$$|r'(t)|^2 = 3^2 \cos^2 t \sin^2 t (\cos^2 t + \sin^2 t)$$

$$= 9(\cos t \sin t)^2$$

$$= 9 \left(\frac{\sin 2t}{2} \right)^2 = \frac{9}{4} \sin^2(2t) = \left(\frac{3}{2} \sin(2t) \right)^2$$

Arclength from 0 to $\frac{\pi}{2}$

$$= \int_0^{\frac{\pi}{2}} |r'(t)| dt = \int_0^{\frac{\pi}{2}} \frac{3}{2} |\sin(2t)| dt$$

$$= \frac{3}{2} \left(\frac{-\cos 2t}{2} \right) \Big|_0^{\frac{\pi}{2}} = \frac{3}{2} \left(-\frac{(-1)}{2} - \frac{-1}{2} \right) = \frac{3}{2} \cdot 1$$

$$= \frac{3}{2}$$

Note: $\sin 2t$ is positive between 0 and $\frac{\pi}{2}$.

So $|\sin 2t| = \sin 2t$.