13.4

5) $r(t) = (2t+3, 5-t^2)$

$v(t) = r'(t) = (2, -2t)$, $|r'(t)| = |2(1,-t)| = 2\sqrt{1+t^2}$

$$T(t) = \frac{v(t)}{|v(t)|} = \left(\frac{1}{\sqrt{1+t^2}}, \frac{-t}{\sqrt{1+t^2}} \right)$$

To find $N(t)$ and $K(t)$, we would usually use the formulae, $N(t) = \dfrac{T'(t)}{|T'(t)|}$ and $K(t) = \dfrac{|T'(t)|}{|v(t)|}$

Here, these formulae lead to long computations and there is an easier way.

$x(t) = 2t+3$, $y(t) = 5-t^2$

So, $t = \dfrac{x-3}{2}$, $y = 5 - \left(\dfrac{x-3}{2}\right)^2 = \dfrac{-x^2}{4} + \dfrac{6x}{4} - \dfrac{9}{4} + 5$

$y = ax^2 + bx + c$ is always a parabola. Here $a < 0$. So the
$(a \neq 0)$

parabola bends downwards. 

For a curve $y = f(x)$, $K(x) = \dfrac{|f''(x)|}{|1 + f'(x)^2|^{3/2}}$

Here, $y' = \dfrac{-x}{2} + \dfrac{6}{4}$, $y'' = \dfrac{-1}{2}$

~~So~~ $= \dfrac{3}{2} - \dfrac{x}{2} = \dfrac{3-x}{2} = -t$

So $K(t) = \left|\frac{-1}{2}\right| \Big/ \left(1 + (-t)^2\right)^{3/2} = \dfrac{1}{2\left(\sqrt{1+t^2}\right)^3}$

$N(t) \perp T(t)$, $|N(t)| = 1$ and $N(t)$ points to the concave side, Here, this means that the y-component of $N(t)$ is negative.

Also, the perpendiculars to (a,b) are $(-b,a)$ and $(b,-a)$. (Check that their dot product with (a,b) is 0).

~~So~~ Since $T(t) = \left(\frac{1}{\sqrt{1+t^2}}, \frac{-t}{\sqrt{1+t^2}} \right)$

The perpendiculars would be, (ignoring the constant)

$(t, 1)$ x

and $(-t, -1)$ ✓ → negative y-component

Since $|N(t)| = 1$ ~~and we~~

we have $N(t) = \frac{(-t, -1)}{|(-t, -1)|} = \left(\frac{-t}{\sqrt{1+t^2}}, \frac{1}{\sqrt{1+t^2}} \right)$