3. For exponential decay of trout, the eqn. would be

$x' = ax \ , \ a < 0$

and the logistic eqn. for bass (see 9.4) would be

$y' = r(M-y)y$

Since trout and bass compete, we do a similar modification to the one in the worked example 9.5

$$x' = ax - bxy \ , \ y' = r(M-y)y - nxy \ , \quad b, n > 0$$

i.e. we add a negative effect proportional to frequency of interactions, which is proportional to xy.

$$x' = 0 \Rightarrow (a - by)x = 0 \Rightarrow x = 0 \text{ or } y = \frac{a}{b}$$

$y = \frac{a}{b}$ isn't possible as $\frac{a}{b} < 0$ and population is always +ve.

$$y' = 0 \Rightarrow \big(r(M-y) - nx\big)y = 0 \Rightarrow y = 0 \text{ or } r(M-y) - nx = 0$$

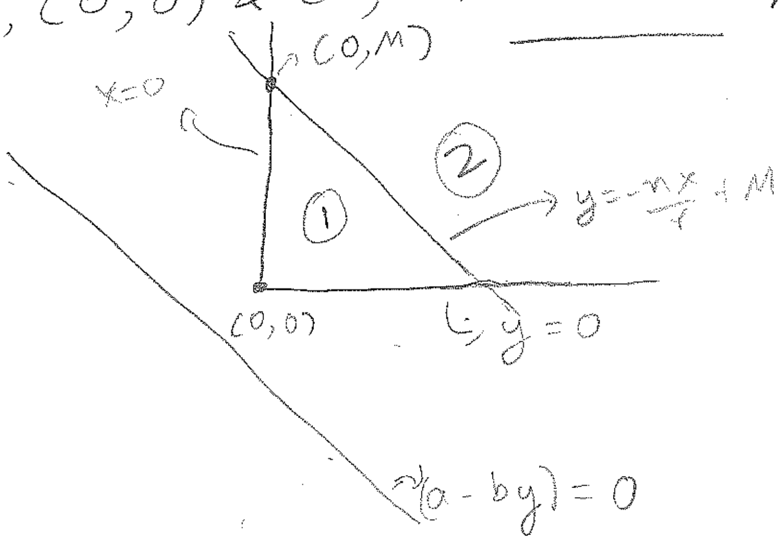
At equilibrium $x' = 0$ and $y' = 0$

So, $x = 0, y = 0$ OR $x = 0, \ r(M-y) - nx = 0$

$\Leftrightarrow r(M-y) - n0 = 0$

$\Leftrightarrow y = M$

So, $(0,0)$ & $(0, M)$ are the equilibria.



$r(M-y) - nx = 0$

$\Leftrightarrow$

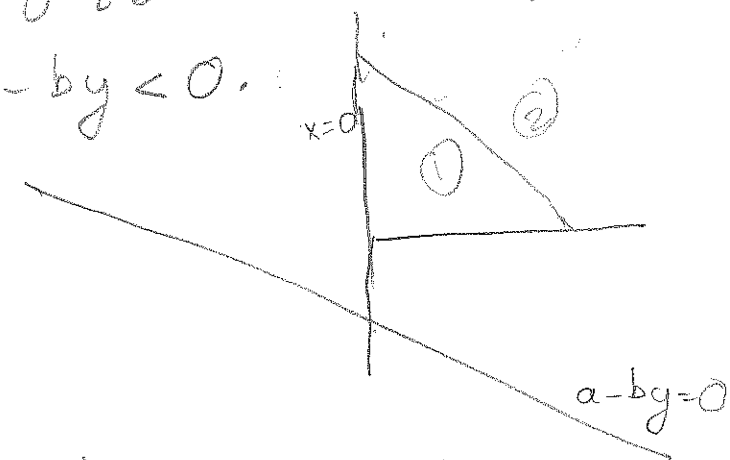
$y = -\frac{nx}{r} + M$

We try to find where x' and y' are +ve/-ve by plotting the boundaries, where x' and y' will be 0.

We are only interested in $x \geq 0$ and $y \geq 0$ as our populations are +ve.

$x' = (a - by)x$. In region ① $x > 0$ and $a - by < 0$ (As you move above the line $a - by = 0$, y increases causing "by" to become larger, so "$a - by$" decreases from 0 to -ve values)

In region ② also $x > 0$, $a - by < 0$.



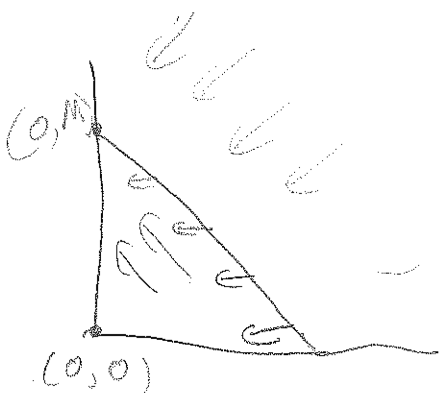
$y' = (r(M-y) - nx)y$

In regions ① & ② $y > 0$

$r(M-y) - nx = 0$ on the line separating ① and ②.

x and y increase as one moves rightwards and upwards, $r(M-y) - nx$ decreases.

So, $r(M-y) - nx < 0$ in ② and > 0 in ①.



in, ①, $\underset{-}{x'} = (a - \underset{-}{by})\ \underset{+}{x}$, $x' < 0$ (West)

$\underset{+}{y'} = (\underset{+}{r(M-y) - nx})\ \underset{+}{y}$, $y' > 0$ (North)

(North-West)

in ②, $\underset{-}{x'} = (a - \underset{-}{by})\ \underset{+}{x}$, $x' < 0$ (West)

$\underset{-}{y'} = (\underset{-}{r(M-y) - nx})\ \underset{+}{y}$, $y' < 0$ (South)

(South-West)

On the bdry. between ① and ② $y' = 0, x' < 0$ (West)

So all solution curves will eventually end up at (0, M), unless one is initially at (0,0).

So (0, M) is a stable equilibria, and (0,0) is unstable.

Coexistence isn't possible as trout population always reaches 0.