

$$y' + P(x)y = Q(x)$$

$$v(x) = e^{\int P(x) dx}$$

← Integrating factor

$$v(x) y' + v(x) P(x) y = v(x) Q(x)$$

see book for explanation

$$(v(x) y)' = v(x) Q(x)$$

$$\int (v(x) y)' = \int v(x) Q(x) \quad \left(\int f' = f \right)$$

$$v(x) y = \int v(x) Q(x)$$

$$y = \frac{\int v(x) Q(x)}{v(x)}$$

Indefinite integral include constant

$$\text{fo9.10) } xy' - 2y = x^2$$

$$y' - \underbrace{\frac{2}{x}}_P y = \underbrace{x}_Q, \quad P(x) = \frac{-2}{x}, \quad Q(x) = x$$

$$v(x) = e^{\int P dx} = e^{\int \frac{-2}{x}} = e^{-2 \ln|x|} = (e^{\ln|x|})^{-2} = x^{-2}$$

$$y = \frac{\int x^{-2} x}{x^{-2}} = \frac{\int x^{-1}}{x^{-2}} = \frac{\ln|x| + C}{x^{-2}}$$

$$y = x^2 (\ln|x| + C)$$

$$y(-1) = 0$$

$$0 = (-1)^2 (\ln|-1| + C)$$

$$0 = 1 (\ln 1 + C) = 1(0 + C)$$

$$0 = C$$

$$\text{So, } y(x) = x^2 \ln|x|$$