Math 114 Quiz 4

Name: ________________________________ Section: ____________

If $x^6 + y^6 = 1$, find the maximum and minimum of $x^4 + y^4$?

Solution:
Use Lagrange multiplier, the target function is

$$f(x, y) = x^4 + y^4,$$

and the restriction is

$$g(x, y) = x^6 + y^6 - 1 = 0.$$

So critical points should satisfy

$$\nabla f = \lambda \nabla g,$$

which gives us two equations:

$$4x^3 = 6\lambda x^5 \tag{1a}$$

$$4y^3 = 6\lambda y^5 \tag{1b}$$

Equation (1a) holds when $x = 0$ or $x^2 = \frac{2}{3\lambda}$, equation (1b) holds when $y = 0$ or $y^2 = \frac{2}{3\lambda}$. Plug each case into the restriction

$$x^6 + y^6 - 1 = 0,$$

we get the following 8 pairs:

$$(0, \pm 1),\ (\pm 1, 0),\ (\pm 2^{-\frac{1}{6}}, \pm 2^{-\frac{1}{6}}).$$

Our target function f takes value 1 at the first 4 pairs, and takes value $\sqrt[3]{2}$ at the other 4 pairs. So 1 is the minimum and $\sqrt[3]{2}$ is the maximum.

Remark: Here we use that a continuous function must has maximum and minimum on a bounded closed region. Since we only find 2 extrema, the larger one must be the maximum and the smaller one must be the minimum.