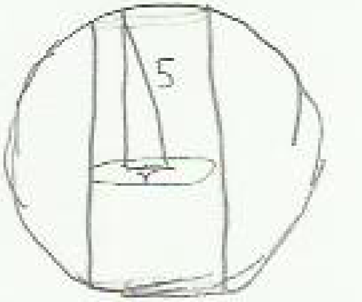
Math 114 Quiz 5

Name: ______________________ Section: ____________

Integrate the function $f(x, y, z) = x^2$ in the region inside the sphere of radius 5 around the origin, and also inside the solid cylinder $x^2 + y^2 = 4$.



Cross-section is a circle of radius 2.

Max. height depends on r

$z_{max} = \sqrt{25-r^2}$, $z_{min} = -\sqrt{25-r^2}$

$$\int_0^{2\pi}\int_0^2\int_{-\sqrt{25-r^2}}^{\sqrt{25-r^2}} (r\cos\theta)^2\, r\, dz\, dr\, d\theta = \int_0^{2\pi}\int_0^2 r^3\cos^2\theta\; 2\sqrt{25-r^2}\, dr\, d\theta$$

$$= \int_0^{2\pi}\cos^2\theta\, d\theta \int_0^2 2r^3\sqrt{25-r^2}\, dr$$

$$\int_0^2 2r^3\sqrt{25-r^2}\, dr = \int \frac{2r^3}{-2r}\sqrt{u}\, du = \int -r^2\sqrt{u}\, du = \int (u-25)\sqrt{u}\, du$$

$u = 25 - r^2$

$$= \int u^{3/2} - 25u^{1/2}\, du = \frac{2u^{5/2}}{5} - \frac{50u^{3/2}}{3} = \frac{2}{5}(25-r^2)^{5/2} - \frac{50}{3}(25-r^2)^{3/2}\Big|_0^2$$

$$= \frac{2}{5}21^{5/2} - \frac{50}{3}21^{3/2} - \left(\frac{2\cdot 5^5}{5} - \frac{50\cdot 5^3}{3}\right) = \frac{2}{5}21^{5/2} - \frac{50}{3}21^{3/2} - 5^4\left(2 - \frac{10}{3}\right)$$

$$\int_0^{2\pi}\cos^2\theta = \int_0^{2\pi}\frac{1+\cos 2\theta}{2}\, d\theta = \frac{\theta}{2} + \frac{\sin 2\theta}{4}\Big|_0^{2\pi} = \pi$$

So, Answer $= \left(\frac{2}{5}21^{5/2} - \frac{50}{3}21^{3/2} + 5^4\cdot\frac{4}{3}\right)\pi$