



Math 114

Quiz 5

Name: \_\_\_\_\_

Section: \_\_\_\_\_

Integrate the function  $f(x, y, z) = z$  in the region above the cone  $z = \sqrt{x^2 + y^2}$  and inside the sphere of radius 5 around the origin.

$$z = \sqrt{x^2 + y^2}, \quad \rho \cos \phi = \rho \sin \phi, \quad \cos \phi = \sin \phi$$
$$z = r$$

$$\tan \phi = 1, \quad \phi = \frac{\pi}{4}$$



$$0 \leq \rho \leq 5, \quad 0 \leq \phi \leq \frac{\pi}{4}, \quad 0 \leq \theta \leq 2\pi$$

$$\int_0^{2\pi} \int_0^{\pi/4} \int_0^5 (\rho \cos \phi) (\rho^2 \sin \phi) d\rho d\phi d\theta$$

$$= \int_0^{2\pi} \int_0^{\pi/4} \left[ \frac{\rho^4}{4} \right]_0^5 \cos \phi \sin \phi d\phi d\theta$$

$$= \frac{625}{4} \int_0^{2\pi} \int_0^{\pi/4} \frac{\sin 2\phi}{2} d\phi d\theta$$

$$= \frac{625}{4} 2\pi \left( \frac{1}{2} \left( \frac{1 - \cos 2\phi}{2} \right) \right) \Big|_0^{\pi/4} = \frac{625}{4} \frac{\pi}{2} (0 - (-1)) = \frac{625\pi}{8}$$