Math 114 Quiz 2

Name: ______________________________ Section: ____________

Consider the plane $Ax + By + Cz = 1$, where $A \neq 0$,
a) Find the distance from (0,0,0) to this plane;
b) Find the angle between this plane and the xy-plane;
c) This plane intersects the xy-plane in a line, find the parametric equation for this line.

Solution:
a) Use the formula from a point to a plane:

$$\frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{|\mathbf{n}|}$$

where Q is the given point $(0,0,0)$ and P is any point on the plane, say $(\frac{1}{A}, 0, 0)$, then $\overrightarrow{PQ} = (-\frac{1}{A}, 0, 0)$, the normal vector of the plane is (A, B, C), plug these into the formula, we get the distance is

$$\frac{|(-\frac{1}{A}, 0, 0) \cdot (A, B, C)|}{\sqrt{A^2 + B^2 + C^2}} = \frac{1}{\sqrt{A^2 + B^2 + C^2}}$$

Remark: If you choose another point on the plane $Ax+By+Cz = 1$, you will get the same result.

b) The xy-plane is the one containing x-axis and y-axis, its equation is $z = 0$, so its normal vector is $(0, 0, 1)$, and the angle between the planes is the angle between the normal vectors, which is

$$\cos^{-1} \frac{(0,0,1) \cdot (A, B, C)}{|(0,0,1)||(A, B, C)|} = \cos^{-1} \frac{C}{\sqrt{A^2 + B^2 + C^2}}.$$

c) First find a point that on both of the planes, we can check $(\frac{1}{A}, 0, 0)$ is such a point. Then find the vector parallel to the line, and we can get this be taking cross product of the normal vector of the planes, so

$$\mathbf{v} = (0, 0, 1) \times (A, B, C) = (-B, A, 0),$$

so the parametric equation is

$$x = \frac{1}{A} - Bt, \ y = At, \ z = 0.$$