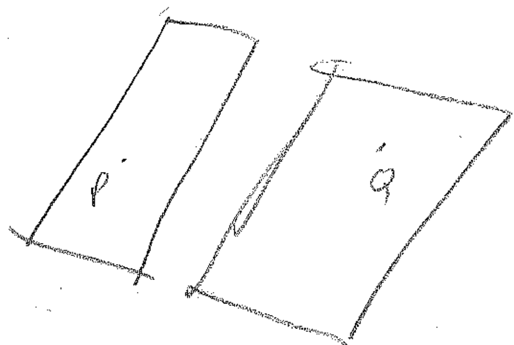
1. Let P be a point on the plane $3x + y - 2z = 2$ and let Q be a point on the plane $3x + y - 2z = 5$. Let $\mathbf{v}$ be the vector that points from P to Q. Which of the following *cannot* be true?

(A) $|\mathbf{v}| = 10$
(B) $|\mathbf{v}| = 100$
(C) $\mathbf{v} \cdot (3\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 0$
(D) $|\mathbf{v} \times (3\mathbf{i} + \mathbf{j} - 2\mathbf{k})| = 0$
(E) $|\mathbf{v} \times (3\mathbf{i} + \mathbf{j} - 2\mathbf{k})| = 100$
(F) The x-coordinate of $\mathbf{v}$ is zero.
(G) The y-coordinate of $\mathbf{v}$ is zero.
(H) The z-coordinate of $\mathbf{v}$ is zero.



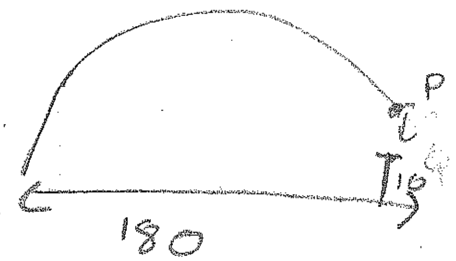
The two planes are parallel as they share a normal vector $(3, 1, -2)$

So, $\overrightarrow{PQ}$ cant be normal to $(3,1,-2)$ as that would mean P, Q lie on same plane.

C)

2. In October 2010, Josh Scobee of the Jacksonville Jaguars kicked a remarkable last-second 60-yard (180 feet) field goal to beat the Indianapolis Colts. The football left the ground with an initial horizontal velocity of 45 feet per second toward the goalpost and an initial upward velocity of 67 feet per second. The crossbar of the goalpost is 10 feet high. By how many feet did the ball clear the crossbar? (Assume the acceleration due to gravity is 32 ft/sec^2 and ignore wind resistance.)

(A) 2 feet	(B) 4 feet	(C) 6 feet	(D) 8 feet
(E) 10 feet	(F) 12 feet	(G) 14 feet	(H) 16 feet



Horizontal velocity = 45 ft/sec

Time to reach P = $\dfrac{180\,ft}{45\,\frac{ft}{sec}}$ = 4 sec

$h(t)$ = height at time t

$h(0) = 0$ (ball starts from ground)

$h'(0) = 67$ (vertical velocity)

$h''(t) = -32$ (gravity)

$h'(t) = -32t + C$, $h'(0) = C = 67$

$h'(t) = -32t + 67$

$h(t) = -\dfrac{32t^2}{2} + 67t + C$, $h(0) = C = 0$

$h(4) = -16 \cdot 4^2 + 67 \cdot 4 + 0 = 12$

Ans = $12 - 10 = 2$ ft

A

3. Let P be the plane tangent to the graph of $x^2e^{2y-z} = 4$ at the point $(2,1,2)$. At what point does P intersect the z-axis?

(A) $(0,0,3)$ (B) $(0,0,2)$ (C) $(0,0,1)$ (D) $(0,0,0)$

(E) $(0,0,-1)$ (F) $(0,0,-2)$ (G) $(0,0,-3)$ (H) $(0,0,-4)$

$f(x,y,z) = x^2 e^{2y-z} = 4$

$f_x = 2x e^{2y-z}$, $f_y = x^2 2e^{2y-z}$, $f_z = -x^2 e^{2y-z}$

at $(2,1,2)$

$f_x = 4$, $f_y = 8$, $f_z = -4$

Eqn of plane

$4(x-2) + 8(y-1) + (-4)(z-2) = 0$

z-axis is $x = y = 0$

So, $-8 + -8 + (-4)z + 8 = 0$

$z = -2$

$(0,0,-2)$ $\underline{\underline{E}}$

4. Calculate the arclength of the curve given parametrically by

$$x = 2t^3, \qquad y = 6t, \qquad z = \frac{3}{t}$$

for $1 \le t \le 3$.

(A) 8 (B) $24\sqrt{3}$ (C) 36 (D) 54

(E) $54\sqrt{3}$ (F) $\frac{136}{3}$ (G) 81 (H) $144\sqrt{3}$

$r'(t) = \langle 6t^2, 6, -\frac{3}{t^2} \rangle$

Arclen $= \int_1^3 |r'(t)| = \int_1^3 \sqrt{36t^4 + 36 + \frac{9}{t^4}}\, dt$

$= \int_1^3 6t^2 + \frac{3}{t^2}\, dt = 54$ $\underline{\underline{D}}$

5. If three resistors, having resistances are R_1 ohms, R_2 ohms and R_3 ohms respectively, are connected in parallel, then the overall resistance R of the circuit satisfies the equation

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

At a moment when $R_1 = 6$ ohms, $R_2 = 3$ ohms and $R_3 = 2$ ohms, we have that R_1 is increasing at a rate of 2 ohms/sec, R_2 is increasing at a rate of 4 ohms/sec and R_3 is increasing at a rate of 6 ohms/sec. How fast is the overall resistance R changing?

(A) 6 ohms/sec (B) 4 ohms/sec (C) 2 ohms/sec (D) 1 ohm/sec

(E) 1/2 ohm/sec (F) 1/3 ohm/sec (G) 1/4 ohm/sec (H) 1/6 ohm/sec

$$\begin{tikzcd} & R_1 \arrow[dr, no head] & \\ t \arrow[ur] \arrow[r] \arrow[dr] & R_2 \arrow[r, no head] & R \\ & R_3 \arrow[ur, no head] & \end{tikzcd}$$

$$\frac{dR}{dt} = \frac{\partial R}{\partial R_1}\frac{dR_1}{dt} + \frac{\partial R}{\partial R_2}\frac{dR_2}{dt} + \frac{\partial R}{\partial R_3}\frac{dR_3}{dt}$$

$$\frac{\partial R}{\partial R_1} = \frac{-1}{C^2}\frac{\partial C}{\partial R_1} \qquad R = \frac{1}{C} \text{ where } C = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= -\frac{1}{C^2}\left(\frac{-1}{R_1^2}\right) = \frac{R^2}{R_1^2}$$

$$R = \frac{1}{C} = \frac{1}{1} \quad \text{as } C = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$$

So $\dfrac{\partial R}{\partial R_1} = \dfrac{1}{6^2} = \dfrac{1}{36}$, $\dfrac{\partial R}{\partial R_2} = \dfrac{R}{R_2^2} = \dfrac{1}{9}$, $\dfrac{\partial R}{\partial R_3} = \dfrac{1}{4}$

$$\frac{dR}{dt} = \frac{1}{36}\cdot 2 + \frac{1}{9}\cdot 4 + \frac{1}{4}\cdot 6 = 2 \text{ ohm/sec}$$

$\underline{\underline{C}}$

6. The function $f(x,y) = 3x^2 + 6xy - y^3$ has

(A) one local maximum and one saddle point
(B) one local minimum and one saddle point
(C) one local maximum and one local minimum
(D) two local maxima
(E) two local minima
(F) two saddle points
(G) a local maximum and no other critical points
(H) a saddle point and no other critical points

$f_x = 6x + 6y, \; f_y = 6x - 3y^2$

$f_x = 0$ and $f_y = 0$ at critical pt.

So, $x = -y$ and $2x = y^2$

$2x = (-x)^2 = x^2, \; x^2 = 2x, \; x = 0$ or 2

Critical points: $(0,0)$ or $(2,-2)$

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{vmatrix} = \begin{vmatrix} 6 & 6 \\ 6 & -6y \end{vmatrix} = -36y - 36$$

$D(0,0) = -36$ So, $(0,0)$ is saddle pt.

$D(2,-2) = -2(-36) > 0$

$f_{xx}(2,-2) = 6 > 0$. So $(2,-2)$ is local min.

B

7. Maximize $f = x + yz$ on $g = x^2 + y^2 + z^2 = 4$

$\nabla f = \lambda \nabla g$

$\langle 1, z, y \rangle = \lambda \langle 2x, 2y, 2z \rangle$

Assume $x, y, z \neq 0$

$\lambda = \frac{1}{2x} = \frac{z}{2y} = \frac{y}{2z}$

$2z^2 = 2y^2$

$z = \pm y$

$\frac{1}{2x} = \frac{z}{2(\pm z)} = \frac{1}{\pm 2}$

So, $x = \pm 1$

$(\pm 1)^2 + y^2 + y^2 = 4$

$2y^2 = 3$

$y = \pm\sqrt{\frac{3}{2}} = z$

$\left(\pm 1, \pm\sqrt{\frac{3}{2}}, \pm\sqrt{\frac{3}{2}}\right)$

f is maximum among these 8 points when all signs are positive

$f\left(1, \sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}}\right) = 1 + \frac{3}{2} = 2\cdot 5$

What if assumption fails

$x \neq 0$ as $\lambda 2x = 1$

If $y = 0$

$z = \lambda 2y = 0$

$y = z = 0$

$x^2 + 0 + 0 = 4$

$x = \pm 2$

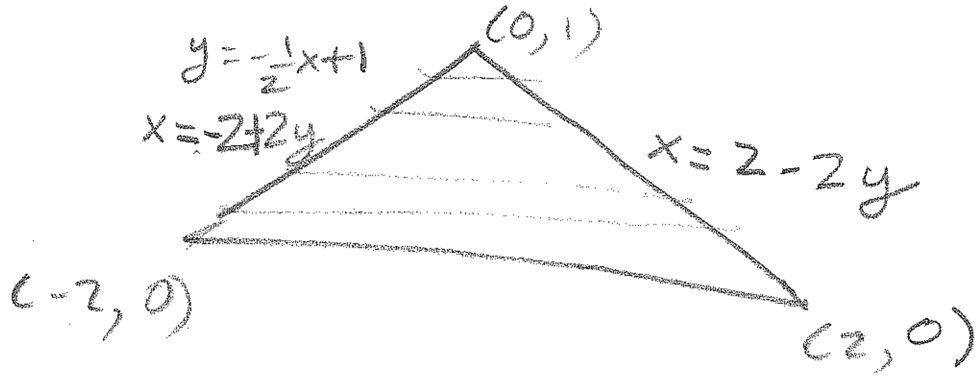
$f(\pm 2, 0, 0) = \pm 2$

So maximum among these 10 points is $+2\cdot 5$ attained at

$\left(1, \sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}}\right)$

G

8.



$$0 \le y \le 1$$

$$-2+2y \le x \le 2-2y$$

$$\int_0^1 \int_{-2+2y}^{2-2y} y \, dx \, dy$$

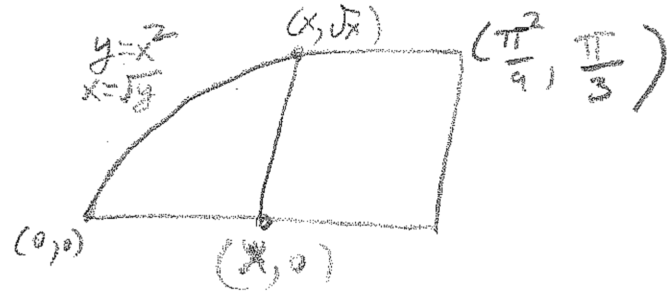
$$= \int_0^1 y\left((2-2y)-(-2+2y)\right) dy$$

$$= \int_0^1 y(4-4y) = \int_0^1 4y - 4y^2 \, dy$$

$$= 4\frac{y^2}{2} - \frac{4y^3}{3}\Big|_0^1 = 2 - \frac{4}{3} = \frac{2}{3}$$

C

9. $\displaystyle\int_0^{\pi/3}\int_{y^2}^{\pi^2/9} \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx\,dy$



$0 \le x \le \pi^2/9$

$0 \le y \le \sqrt{x}$

$$\int_0^{\pi^2/9}\int_0^{\sqrt{x}} \frac{\sin\sqrt{x}}{x}\,dy\,dx = \int_0^{\pi^2/9} \frac{\sin\sqrt{x}}{x}\, y\Big|_0^{\sqrt{x}}\,dx$$

$$= \int_0^{\pi^2/9} \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = \int_0^{\pi/3} \frac{\sin u}{u}(2du\,u)$$

$u=\sqrt{x}$

$\frac{du}{dy} = \frac{1}{2\sqrt{x}}$

$dx = 2u\,du$

$0 \le x \le \pi^2/9$

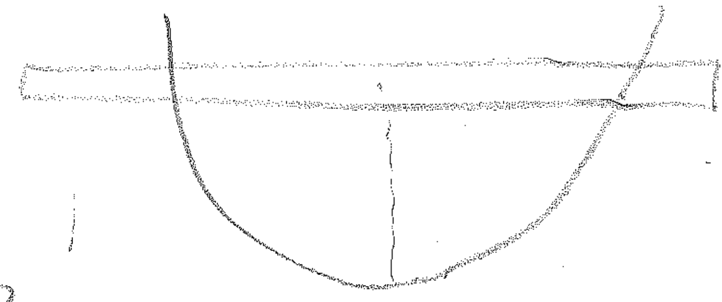
$0 \le u \le \pi/3$

$$= \int_0^{\pi/3} 2\sin u\,du = -2\cos\frac{\pi}{3} - (-2)$$

$= -1 + 2$

$= 1$

10.



Intersection:

$z = 1 = e^{x^2+y^2-4}$

$e^0 = 1, \quad x^2+y^2-4=0$

Circle of radius 2

$$0 \le r \le 2$$
$$0 \le \theta \le 2\pi$$
$$e^{r^2-4} \le z \le 1$$

$$\int_0^2 \int_0^{2\pi} \int_{e^{r^2-4}}^{1} 1 \; r \, dz \, d\theta \, dr$$

$$= \int_0^2 \int_0^{2\pi} \left(1 - e^{r^2-4}\right) r \, d\theta \, dr$$

$$= 2\pi \int_0^2 r - e^{r^2-4} r \, dr$$

$$= 2\pi \left(\frac{r^2}{2}\Big|_0^2 - \frac{1}{2}(1-e^{-4}) \right) = 2\pi \left(2 - \frac{1}{2} + \frac{e^{-4}}{2}\right)$$

$$= \pi\left(3 + e^{-4}\right)$$

$$\int_0^2 r e^{r^2-4} \, dr \qquad u = r^2$$

$$= \int_0^4 \frac{e^{u-4}}{2} \, du$$

$$= \frac{1}{2}\left(1 - e^{-4}\right)$$

H

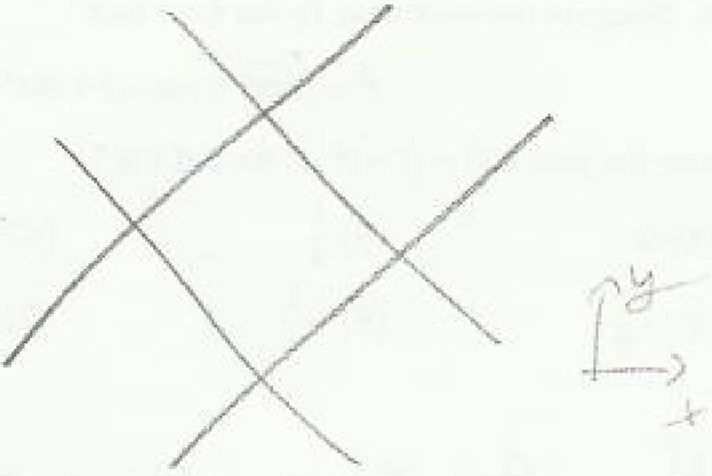
11 $\iint x+2y \, dA$

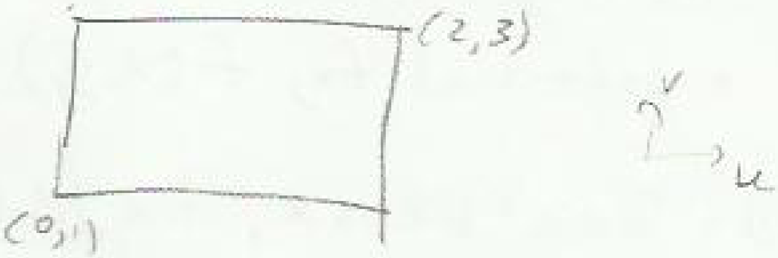
$u = 2x+y$

$v = x-y$

$0 \le u \le 2$

$1 \le v \le 3$





$du\,dv = \left|\dfrac{\partial(u,v)}{\partial(x,y)}\right| dx\,dy$

$= \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}$

$= |-3| = 3$, $dx\,dy = \dfrac{du\,dv}{3}$

$x+2y = u-v$

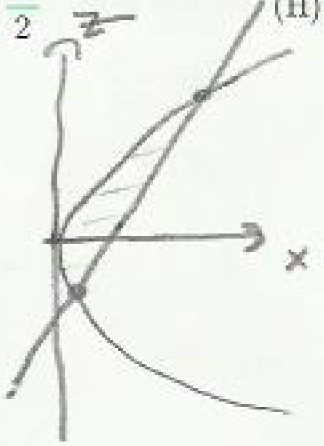
$\displaystyle\int_0^2\int_1^3 (u-v)\frac{dv\,du}{3} = \left(2\frac{u^2}{2}\Big|_0^2 - 2\frac{v^2}{2}\Big|_1^3\right)\frac{1}{3} = -\frac{4}{3}$

B

12. Compute the volume of the solid bounded by the four surfaces $x = z^2$, $x = 4 + 3z$, $x + y = 0$ and $x + y = 1$

(A) $\frac{8}{3}$ (B) $\frac{13}{2}$ (C) $\frac{22}{3}$ (D) $\frac{25}{2}$

(E) 12 (F) $\frac{35}{3}$ (G) $\frac{27}{2}$ (H) $\frac{50}{3}$



Intersection in xz plane

$x = z^2 = 4+3z$

$z^2 - 4 - 3z = 0$

$(z+1)(z-4) = 0$

$z = -1,\ x = 1;$ or $z = 4,\ x = 16$

$-1 \le z \le 4$

$z^2 \le x \le 4+3z$

$-x \le y \le 1-x$

$$\int_{-1}^{4}\int_{z^2}^{4+3z}\int_{-x}^{1-x} 1\,dy\,dx\,dz = \int_{-1}^{4}\int_{z^2}^{4+3z} 1-x+x\,dx\,dz$$

$$= \int_{-1}^{4} 4+3z-z^2\,dz = \frac{125}{6}$$

not any of the choices

13. Compute the work done by the force field

$$\vec{F} = (8xe^y + \cos x)\,\mathbf{i} + (4x^2e^y + 4y^3)\,\mathbf{j}$$

along the path $\mathbf{c}(t) = (t - t^2\,,\, t)$ for $0 \leq t \leq 1$.

(A) 0	(B) $\frac{1}{4}$	(C) 1	(D) -1
(E) $-\frac{1}{2}$	(F) $-\frac{1}{4}$	(G) -2	(H) 2

$N_x = 8xe^y + 0$, $M_y = 8xe^y + 0$

$N_x = M_y$, So $\vec{F}$ is conservative, we can find a potential fn, $f(x,y)$

$f_x = M = 8xe^y + \cos x$, $f = \int M\,dx = 4x^2e^y + \sin x + g(y)$

Constant is fn of y

$f_y = 4x^2e^y + 0 + \frac{dg}{dy} = N = 4x^2e^y + 4y^3$

$\frac{dg}{dy} = 4y^3$, $g(y) = y^4 + C$

So $f(x,y) = 4x^2e^y + \sin x + y^4 + C$

$f(c(1)) = f(0,1) = \underline{1} + C$

$f(c(0)) = f(0,0) = 0 + C$

Ans $= f(c(1)) - f(c(0)) = \underline{1}$ $\underline{C}$

14. Evaluate

$$\int_C (12x^2y^5 + 2x)\,dx + (20x^3y^4 + 3x)\,dy$$

where C is the unit circle $x^2 + y^2 = 1$, traversed counter-clockwise.

(A) 0 (B) π (C) $\frac{\pi}{2}$ (D) 2π

(E) $\frac{\pi}{3}$ (F) 3π (G) $\frac{\pi}{4}$ (H) 4π

$$\int_C M\,dx + N\,dy = \int_0^{2\pi}\int_0^1 (N_x - M_y)\, r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^1 60x^2y^4 + 3 - 60x^2y^4 \;(r\,dr\,d\theta)$$

$$= \int_0^{2\pi}\int_0^1 3\,(r\,dr\,d\theta) \quad = 3\pi$$

F

/

15. $t\dfrac{dy}{dt} = t^{5/2} + 2y$

Standard form:

$y' + P(t)\,y = Q(t)$

Integrating factor

$v(t) = e^{\int P(t)\,dt}$

Soln

$y(t) = \dfrac{1}{v(t)}\int v(t)\,Q(t)$

$y' = t^{3/2} + \dfrac{2y}{t}$

$y' - \left(\dfrac{2}{t}\right)y = t^{1.5}$

$v(t) = e^{\int -\frac{2}{t}\,dt}$

$= e^{-2\ln|t|}$

$= \dfrac{1}{|t|^2} = \dfrac{1}{t^2}$

$y(t) = \dfrac{1}{1/t^2}\int \dfrac{1}{t^2}\,t^{1.5} = t^2\int t^{-0.5}\,dt$

$= t^2\left(\dfrac{t^{0.5}}{0.5} + C\right) = 2t^{2.5} + C\,t^2$

$y(1) = 3$

$y(1) = 2\,1^{2.5} + C = 2 + C = 3, \quad C = 1$

$y(t) = 2\,t^{2.5} + t^2$

$y(4) = 2\,4^{2.5} + 4^2 = 2\cdot 16\cdot\sqrt{4} + 16 = 80$

E