UNIVERSITY *of* PENNSYLVANIA
MATHEMATICS DEPARTMENT
Mathematics 114 – Midterm I
Spring 2004

Your Name:_____ **Penn *ID#*_____**

You have 1.5 hours to complete this examination.

Please show all work in the space provided on your test paper and write your answers in the appropriate place below.

DO NOT DETACH THIS SHEET FROM YOUR TEST

Part I: True or (False and Why)

	T/F	***Why?***
1		
2		
3		
4		
5		

Part II: Multiple Choice

1		4		7		10	
2		5		8			
3		6		9			

Part 3: Free Response

1.	***3.*** **a)** **b)** **c)**
2.	***4.***

----------*Please do not write below this line*----------

Scores:

I _____ **II.** _____ **III.** _____

Raw _____ **Scaled** _____

Part I: *True* or (*False and Why*)

- If a statement is *true*, label it as true and do nothing else.
- If a statement is *false*, label it false and, in the space provided, justify your answer. Please note that you are to provide a theorem, definition or counterexample showing why the statement is false; simply correcting the statement so that it is true is *not* considered an adequate answer.
- Each question is worth 4 points.

1. If $\mathbf{w} \neq \mathbf{0}$ **and** $\mathbf{v} \neq \mathbf{0}$ and $\mathbf{w} = k\mathbf{v}$ where k is a scalar, then $\mathbf{v} \times \mathbf{w} = \mathbf{0}$.	1.
2. If **A**, **B** and **C** are three distinct unit vectors and $(\mathbf{A} \times \mathbf{B}) \bullet \mathbf{C} = 0$, then the vectors all lie in the same plane.	2.
3. Suppose **a**, **b** and **c** are three distinct unit vectors. If $\mathbf{b} \times \mathbf{c} \neq \mathbf{0}$ and $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{0}$, then **a** is parallel to **c**.	3.
4. Suppose a curve $y = f(x)$ can be parameterized by $x(t) = e^t$ and $y(t) = e^{2t}$, $0 \leq t \leq \infty$. This parameterization is unique.	4.
5. Suppose the parametric curve $x = f(t)$, $y = g(t)$ satisfies $g'(1)=0$, then the curve has a horizontal tangent at $t = 1$.	5.

Part II: Multiple Choice

- This part contains 10 multiple-choice questions. Please write the letter corresponding to your answer on the appropriate line on your answer sheet. If you change an answer, please either erase or cross out the answer you do not want considered; questions with more than one answer will be marked wrong.
- Please show your work in the space provided on this question sheet. *Little or no credit* will be given for an answer with no supporting work, even if correct. Partial credit *may* be given based on your work.
- Each question is worth 5 points.

1. For what values of a and b will the vector $a\mathbf{i} + b\mathbf{j}$ be orthogonal to the vector $b\mathbf{i} - a\mathbf{j}$?

a) a=3, $b = 3$ only
b) a =3, $b = 3$ or $a = -3$, $b = -3$ only
c) whenever $a = b$
d) whenever $a = -b$
e) for all values of a and b
f) no such values of a and b exist

2. Find a unit vector in the direction of $9\mathbf{i} - 20\mathbf{j} + 12\mathbf{k}$

a) $\frac{9}{25}\mathbf{i} - \frac{4}{5}\mathbf{j} + \frac{12}{25}\mathbf{k}$
b) $9\mathbf{i} - 20\mathbf{j} + 12\mathbf{k}$
c) $\frac{1}{\sqrt{3}}\mathbf{i} - \frac{1}{\sqrt{3}}\mathbf{j} + \frac{1}{\sqrt{3}}\mathbf{k}$
d) $\frac{9}{41}\mathbf{i} - \frac{20}{41}\mathbf{j} + \frac{12}{41}\mathbf{k}$
e) $\frac{1}{9}\mathbf{i} - \frac{1}{20}\mathbf{j} + \frac{1}{12}\mathbf{k}$
f) $\mathbf{i} - \mathbf{j} + \mathbf{k}$

3. Find the length of the curve whose parametric equations are $x = \cos^2 t$, $y = \sin^2 t$, $0 \le t \le \pi$.

a) $\frac{\pi}{2}$ b) $2\sqrt{2}$ c) $4\sqrt{2}$ d) $\frac{\pi}{4}$ e) $\frac{1}{\sqrt{2}}$ f) π

4. Find the smallest positive value of θ for which the curve $r = 1 + \cos\theta$ has a horizontal tangent.

a) $\frac{\pi}{6}$ b) $\frac{2\pi}{3}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{3}$ e) $\frac{3\pi}{4}$ f) $\frac{\pi}{4}$

5. Let l and l' be two lines in space given by $l = \begin{cases} x = 3 + t \\ y = 1 - t \\ z = 2t \end{cases}$; $l' = \begin{cases} x = -1 + t \\ y = 2t \\ z = 1 + kt \end{cases}$. For what value of k are the two lines perpendicular?

a) 0 b) 2 c) $\frac{1}{2}$ d) 1 e) –1 f) no value of k works

6. Find the area of the surface obtained by rotating the curve $x = \sin t$, $y = \sin^2 t$ about the y-axis.

a) $\frac{4\pi\left(\sqrt{\frac{125}{16}} + 1\right)}{3}$ b) $\frac{4\pi\left(\sqrt{\frac{125}{64}} - 1\right)}{3}$ c) $\frac{\pi\left(\sqrt{\frac{125}{64}} - 1\right)}{3}$

d) $\frac{\pi\left(\sqrt{125} - 1\right)}{3}$ e) $\frac{4\pi\left(\sqrt{125} + 1\right)}{3}$ f) $\frac{2\pi\left(\sqrt{\frac{125}{16}} - 1\right)}{3}$

7. For what value(s) of x will the vectors $\mathbf{v} = \langle 2x, 3, x \rangle$ and $\mathbf{w} = \langle 4, 5, x \rangle$ be parallel?

a) 0 b) 0, 2 c) 1.2 d) 0, 1.2 e) 1.2, 2 f) no such x

8. If $\mathbf{a} = \mathbf{i} + 2\mathbf{j}$, $\mathbf{b} = 3\mathbf{j} + 4\mathbf{k}$ and $\mathbf{c} = 5\mathbf{i} + 6\mathbf{k}$, compute the triple scalar product $\mathbf{a} \bullet (\mathbf{b} \times \mathbf{c})$.

a) 58 b) –58 c) 6 d) –5 e) 40 f) 5

9. Find a unit vector orthogonal to the plane determined by the three points (1, 0, 0), (0, 1, 0) and (0, 2, 2).

a) $\left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$ b) $\left\langle \frac{2}{3}, \frac{2}{3}, \frac{-1}{3} \right\rangle$ c) $\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right\rangle$

d) $\left\langle \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$ e) $\left\langle \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0 \right\rangle$ f) $\left\langle \frac{-2}{3}, \frac{1}{3}, \frac{2}{3} \right\rangle$

10. Find the slope of the tangent to the curve with parametric equations $x = 2\ln t$, $y = te^t$ at the point where $t = 1$.

a) 1 b) e c) 2 d) $2e$ e) 3 f) $3e$

Part III: Free Response Problems

- This part contains four questions. Please write your answers on your answer sheet.
- Questions 1 and 2 are worth 5 points each; questions 3 and 4 are worth 10 points each.

1. Let **a**, **b** and **c** be three vectors in the plane $3x - 5y + 6z = 7$. Compute $(-\mathbf{a} + 4\mathbf{b} - 7\mathbf{c}) \bullet (-3\mathbf{i} + 5\mathbf{j} - 6\mathbf{k})$.

2. We know that, in general, $\mathbf{a} \times \mathbf{b} \neq \mathbf{b} \times \mathbf{a}$. Under what special conditions, if any, is $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{a}$? For each set of conditions you find, list the condition(s) and give the value of $\mathbf{a} \times \mathbf{b}$.

3. Consider the polar curve $r = \dfrac{1}{\cos\theta - \sin\theta}$, $0 \le \theta < \dfrac{\pi}{4}$.

a) What happens to r as $\theta \to \dfrac{\pi}{4}$? *Justify your answer.*

b) What value of θ corresponds to the point (1, 0) in Cartesian coordinates?

c) Convert the polar equation to an equation of the form $y = f(x)$.

4. Find the area of the region inside the circle $r = 3\cos\theta$ and outside the cardiod $r = 1 + \cos\theta$.

a) $\frac{\pi}{3}$ b) $\frac{\pi}{2}$ c) π d) $\frac{4\pi}{3}$ e) $\frac{3\pi}{2}$ f) 2π