University *of* Pennsylvania
Mathematics Department
Mathematics 114 – Midterm II
Spring 2004

Your Name:________________________ **Penn *ID#***_____________________

Professor (Check one): Crotty ❑ 001 ❑ 601 Pemantle ❑ 002 Stovall ❑ 003 ❑ 004

You have 1.5 hours to complete this examination.
Please show all work in the space provided on your test paper and write your answers in the appropriate place below.

DO NOT DETACH THIS SHEET FROM YOUR TEST

Part I: True or False

1		6	
2		7	
3		8	
4		9	
5		10	

Part II: Multiple Choice

1		6	
2		7	
3		8	
4		9	
5		10	
6		12	

Part III: Free Response

1.
Maximum Volume: ________________________ **λ =** ____

Significance of λ:

2.

Value: _____________

3.
a) ____

b) ____

4. **min:**________________ **at: (_____, _____)**

max:________________ **at: (_____, _____)**

--*Please do not write below this line*--

Scores:

I _____________ **II.** _____________ **III.** _____________

Raw _____________ **Scaled** ____________

Part I: *True* or *False*

If a statement is *true*, label it as true; if a statement is *false*, label it false and *briefly* explain why.
Don't forget to transfer your answers to the answer sheet.

1. If $f(x, y) \to L$ as $(x, y) \to (a, b)$ along every straight line passing through (a, b), then $\lim_{(x,y)\to(a,b)} f(x,y) = L$	1.
2. If f has a local maximum at (a, b) and if f is differentiable at (a, b), then $\nabla f(a, b) = 0$.	2.
3. If (2, 1) is a critical point of f and $f_{xx}(2,1)f_{yy}(2,1) < [f_{xy}(2,1)]^2$, then f has a saddle point at (2, 1).	3.
4. If $f(x, y) = \sin x + \sin y$, then $-\sqrt{2} \le D_{\hat{\mathbf{u}}} f(x,y) \le \sqrt{2}$.	4.
5. If $f(x, y)$ has two local minima, then f must also have a local maximum.	5.
6. If $\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$, then $f(x, y)$ is differentiable at (a, b).	6.
7. If $\mathbf{u}(t)$ is a vector function of constant length, then $\mathbf{u}(t)$ and its derivative $\mathbf{u}'(t)$ are parallel.	7.
8. If $f(x, y)$ and its partial derivatives f_x, f_y, f_{xy} and f_{yx} are defined and continuous throughout an open region containing the point (a, b), then $f_{xy}(a, b) = f_{yx}(a, b)$.	8.
9. In general, the order of integration in a multiple integral has no effect on the results of the integration; i.e., $\iint_R f(x,y)\,dxdy = \iint_R f(x,y)\,dydx$ for example.	9.
10. Suppose you want to integrate $f(x, y) = \sin x \cos y$ over the region bounded by $x = 0$, $y = 0$ and the parabola $y = 1 - x^2$. One possible integral is $\int_0^{1-x^2} \int_0^1 \sin x \cos y \; dxdy$	10.

Part II: Multiple Choice

Work each problem in the space provided; show sufficient detail that your method of solution and computations are clear. Answers with no supporting work will receive no credit. Write the letter of your choice on the appropriate line of your answer sheet.

1. Let $f(x, y) = x\sin y$. Find $f_{xy}(2, \pi/3)$.

a) $\sqrt{3}$ b) 1 c) $\frac{\sqrt{3}}{2}$

d) 2 e) $\frac{1}{2}$ f) 0

2. What is the domain of the function $z = \sqrt{x^2 + y^2 - 1}$?

a) all real x, y
b) all points on the circle $x^2+y^2=1$
c) all points on and interior to the circle $x^2+y^2=1$
d) all points on and exterior to the circle $x^2+y^2=1$
e) all points such that $y^2 = x^2$
f) all real x, y and z

3. Evaluate $\iint_R f(x,y)\ dA$ if $f(x,y) = x^2 + 2xy$ and $R = \{(x,y)\ \mid\ 0 \le x \le 3,\ 0 \le y \le 4\}$

a) 33 b) 54 c) 136 d) 0 e) 2 f) 5π/2

4. Evaluate the limit $\lim_{(x,y)\to(0,0)} \frac{x^2 y}{\sqrt{x^2 + y^2}}$.

a) 6 b) 4 c) 2 d) 0 e) –2 f) Does Not Exist

5. Find the length of the curve $\mathbf{r}(t) = < \sin 2t,\ \cos 2t,\ 2t^{3/2} >, 0 \le t \le 1$.

a) 13/9 b) 16/9 c) $\dfrac{13\sqrt{13}-6}{27}$ d) $\dfrac{10}{27}\left(\sqrt{13}-2\right)$ e) $\dfrac{2}{27}\left(13\sqrt{13}-8\right)$ f) 22/9

6. Find an equation of the plane tangent to the surface $z = e^{2x+2y}$ at the point (0, 0, 1).

a) $z = 4x + 4y - 7$ b) $z = ex + ey - 2e + 1$ c) $z = e^2x + e^2y - 2e^2 + 1$

d) $z = x + y + 1$ e) $z = 2x + 2y + 1$ f) $z = 4ex + 4ey - 8e + 1$

7. Find $\mathbf{r}(1)$ if $\mathbf{r}'(t) = t^2\mathbf{i} + t^3\mathbf{j}$ and $\mathbf{r}(0) = \mathbf{i}$.

a) $\frac{1}{2}\mathbf{i} + \frac{1}{4}\mathbf{j}$ b) $\frac{4}{3}\mathbf{i} + \frac{1}{4}\mathbf{j}$ c) $\frac{2}{3}\mathbf{i} + \frac{1}{4}\mathbf{j}$ d) $\mathbf{i} + \frac{1}{4}\mathbf{j}$ e) $\frac{1}{3}\mathbf{i} + \frac{3}{4}\mathbf{j}$ f) $\frac{2}{3}\mathbf{i} + \frac{3}{4}\mathbf{j}$

8. Let $z = xy^2 + x^3y$ and let $x = x(t)$ and $y = y(t)$ such that $x(1) = 1$, $y(1) = 2$, $x'(1) = 3$ and $y'(1) = 4$. Find $\frac{dz}{dt}$ when $t = 1$.

a) 42 b) 44 c) 46 d) 48 e) 50 f) 52

9. The position function of a particle is given by $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j}$. Find the normal component of the acceleration vector when $t = 1$.

a) $\frac{1}{\sqrt{5}}$ b) $\frac{2}{\sqrt{5}}$ c) $\frac{3}{\sqrt{5}}$ d) $\frac{4}{\sqrt{5}}$ e) $\frac{5}{\sqrt{5}}$ f) $\frac{6}{\sqrt{5}}$

10. Find the directional derivative of $f(x,y,z) = xe^{xy/z}$ in the direction of $\mathbf{v} = -3\mathbf{i} + 6\mathbf{j} + 6\mathbf{k}$ at the point (3, 0, 1).

a) 17/3 b) 6 c) 19/3 d) 7 e) 22/3 f) 8

11. Given: $x^3 + y^3 = x^2y^2$, find $\frac{dy}{dx}$ when $(x, y) = (2, 2)$.

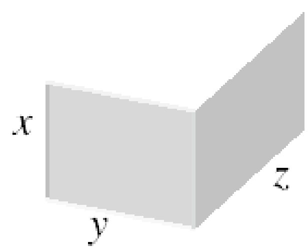
a) –1/3 b) 1/3 c) –1/2 d) 1/2 e) –1 f) 1

12. Evaluate $\iiint_R x^2 \; dV$ where $R = \{(x, y, z) \mid 0 \le x \le y,\ 0 \le y \le 1,\ 0 \le z \le 1\}$.

a) 1/6 b) 1/4 c) 1/3 d) 1 e) 1/2 f) 1/12

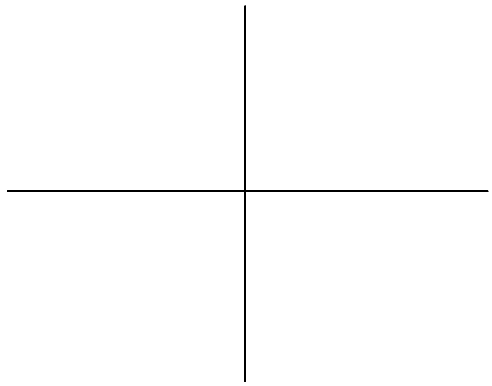
Part III: Free Response Problems (32 points)
Work each problem in the space provided; ***transfer your answers to your answer sheet***.

1. In Lower Slobovia, postal regulations state that the length plus girth of a package ($2x + 2y + z$, see diagram) must not exceed 3 meters. What is the volume ($V = xyz$) in cubic meters of the package of largest volume that can be sent through the mail? What is the value of λ (the Lagrange multiplier) and what is the significance of this value? (Write the value of λ on your answer sheet; include your *brief* description of the significance of this value on this page.)



2. Consider the integral: $\int_0^1 \int_0^{\sqrt{1-x^2}} e^{-(x^2+y^2)}\, dydx$.

a) Sketch the region of integration (be sure to label the axes appropriately):

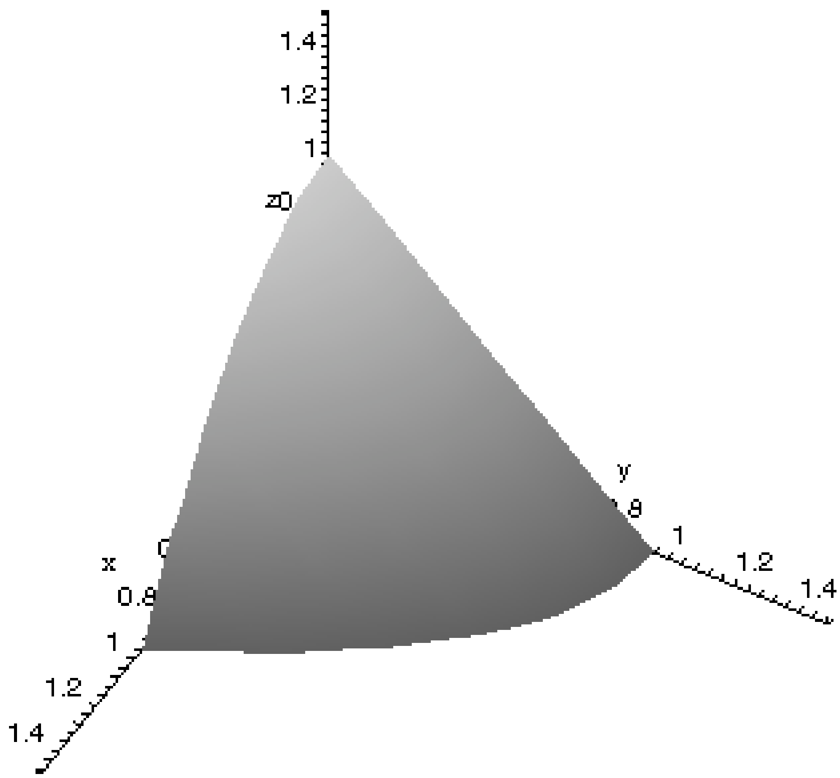


b) Evaluate the integral:

3. Let S be the surface defined by $z = 1 - y - x^2$. Let V be the volume of the three dimensional region bounded by S and the coordinate planes (see diagram below). Set up, but DO NOT EVALUATE, integrals for V:

   a) integrate first with respect to x then with respect to y

   b) integrate first with respect to y then with respect to x



4. Find the absolute maximum and minimum values of $f(x, y) = 3 + xy - x - 2y$ over the closed triangular region R whose vertices are (1, 0), (5, 0) and (1, 4) {see diagram (not to scale!)}.