Math 312 - HOMEWORK 10 - Due Friday, April 24, 2015

Due by 4pm in your grader Yu Wang's mailbox in DRL 4W1.

Each Problem is worth 20 points, Five Problems will be graded. You will also receive up to 10 points per assignment for completeness!

All problems are from the book: Gilbert Strang, Introduction to Linear Algebra, Fourth Edition

Given the matrix A:

$$A = \begin{pmatrix} 5 & -2 & -3 \\ -1 & 4 & -3 \\ 1 & -4 & 3 \end{pmatrix}$$

1. Compute all the eigenvalues of A and write down their algebraic multiplicities.
2. Compute eigenvectors corresponding to the eigenvalues of A: what are the geometric multiplicities of the eigenvalues?
3. Is A diagonalizable? If yes, write it as SDS^{-1}. Otherwise, explain why we can't find S and D.
4. Compute the Jordan decomposition of A... that is, find matrices S and J so that S is invertible, J is in Jordan form, and $A = SJS^{-1}$.
5. Consider a $k \times k$ Jordan block

   $$M = \begin{pmatrix} \lambda & 1 & 0 & \cdots & 0 \\ 0 & \lambda & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & \lambda & 1 \\ 0 & \cdots & \cdots & 0 & \lambda \end{pmatrix}$$

   In words M has λ as every diagonal entry, ones above the diagonal, and all other entries are zero.

   Show that if $M^2 = M$ then k must equal 1, and λ must be either 0 or 1. (**Hint**: Assume $k = 2$ and compute M^2 for an arbitrary λ, and set it equal to M. The argument for general k is quite similar!).
6. Strang. Chapter 8. Section 8.4: # 1, 2.

   Reading: Read through Chapters 6, 7 and 8 of Strang.