Practice Midterm II, Math 312 will NOT be collected or graded

1. (20 points) For each of the following statements, mark whether they are true or false. No work needs to be shown for this problem. (5 problems, 4 points each)

   (a) The matrix $A^T A$ is invertible if and only if the matrix A has linearly independent columns

   (T) **(F)**

   (b) A matrix Q with orthonormal columns satisfies $Q^T Q = -I$.

   (T) **(F)**

   (c) If A is invertible then $\det A = 0$. If A is singular then $\det A \neq 0$.

   (T) **(F)**

   (d) If $det(A) = 1$ for some square matrix A, then there is some b for which $Ax = b$ has infinitely many solutions.

   (T) **(F)**

   (e) The determinant is the unique function which maps $n \times n$ matrices into the real numbers and satisfies the normalization, antisymmetry and multilinearity properties.

   (T) **(F)**

2. (20 points) A subspace V of $\mathbb{R}^3$ is spanned by the columns of

$$A = \begin{pmatrix} 1 & 1 \\ -1 & 0 \\ 1 & 1 \end{pmatrix}$$

(a) Apply the Gram-Schmidt process to find two orthonormal vectors u_1 and u_2 which also span V.

(b) Find an orthogonal matrix Q so that QQ^T is the matrix which orthogonally projects vectors onto V.

(c) Find the best possible (i.e., least squared error) solution to the linear system

$$Q\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

3. (20 points) Given

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$$

(a) Find the matrix P which projects onto the column space of A.

(b) Compute the projection p of b onto this column space.

(c) Find the error $e = b - p$ and show that it lies in the left nullspace of A.

4. (20 points) Calculate the determinant of the matrix A:

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 2 & 6 & 4 & 8 \\ 3 & 1 & 1 & 2 \end{pmatrix}$$

5. (20 points) If you know all 16 cofactors of a 4 by 4 invertible matrix A, how would you find its determinant $det(A)$?