Penn Math Day

Date: July 4th, 2015

Location: Penn Wharton China Center, World Financial Center, West Building 16F, Beijing

Program

10:45–11:45	Alexander Kirillov	*The orbit method and representation theory*
11:45–13:30		Lunch Break
13:30–14:30	Jame Haglund	*The (q,t)-Catalan Numbers in Combinatorics and Geometry*
14:50–15:50	Ching-Li Chai	*Moduli spaces, Torelli loci and special subvarieties*
15:50-17:30		Reception and Discussion

Abstracts.

A. KIRILLOV, *The orbit method and representation theory*.

My talk is an attempt to explain in one-hour the ideas behind the orbit method in representation theory and the geometric quantization in mathematical physics. Representation theory is the mathematical instrument to study and use the symmetry in nature. Geometric quantization is one of possible approaches to construct a quantum physical system from its classical counterpart. The main objects of representation theory are:
(a) unitary irreducible representations of a given group G,
(b) the topological structure of $\hat{G}$, the set of equivalence classes of all unitary irreducible representations, and
(c) explicit description of the restriction and induction functors, infinitesimal and distributional character formulae.

For nilpotent Lie groups all these problems can be naturally answered in terms of the so-called *coadjoint orbits* of G, i.e. the G-orbits in the vector space $\mathfrak{g}^*$, dual to the Lie algebra $\mathfrak{g}$ of G. These answers also make sense for general Lie groups and even beyond, though sometimes with more or less evident corrections. The realization of this program has taken much time and is still not accomplished despite the efforts of many authors.

C.-L. CHAI, *Moduli spaces, Torelli loci and special subvarieties*.

The idea of studying closed Riemann surfaces through their associated abelian varieties, called the Jacobians, goes back to Riemann. Every closed Riemann surface is uniquely determined by its Jacobian variety according to Torelli's theorem. Coleman conjectured that for every $g \geq 4$, there are only a finite number of closed Riemann surfaces of genus g whose Jacobians admit sufficiently many complex multiplications. We will follow this thread and offer a glimpse, from this limited perspective, of the vast enterprise of the exploration of moduli spaces initiated by Riemann. This talk is aimed at students and non-experts; a few open questions are included for the young and intrepid.

J. HAGLUND, *The (q,t)-Catalan Numbers in Combinatorics and Geometry.*

The Catalan numbers are perhaps the most famous sequence in combinatorics, with over 200 known interpretations. In the early 1990's Garsia and Haiman introduced a polynomial in two parameters q,t which reduces to the Catalan number when q and t are equal to 1. In this talk we discuss some of the interpretations for these q,t-Catalan numbers, from representation theory to knot invariants.