Math 180: Course Summary  
Fall 2012, Prof. Jauregui

Below is an outline of most of the topics we covered this semester. In **bold** I have indicated those that I think are most significant to the “take-home message” of this course.

- **Probability**
  - terminology: experiments, outcomes, events, sample space
  - operations: OR, AND, NOT
  - mutually exclusive events
  - independent events, multiplication rule
    * Case: *People v. Collins*
  - rules of probability
    * addition rule, inclusion-exclusion
    * subset rule, complement rule
  - birthday problem – 23 people in a room
  - fifty-fifty fallacy
  - multiple lottery winners
  - “law of averages” fallacy
  - **Conditional probability**
    * multiplication rule for conditional probabilities
  - **Bayes’ formula**
    * disease testing, true/false negatives/positives
    * general misunderstanding of conditional probability
    * applications to DNA matching, determination of guilt
  - Prosecutor’s fallacy, Sally Clark case
  - **Defense attorney’s fallacy**, DNA matching, O.J. Simpson trial
  - Monty Hall problem
  - Expected value, standard deviation
    - Bernoulli trials – repeated experiments

- **Statistics**
  - bell-shaped curve/normal distribution, interpretation of \( \mu \) and \( \sigma \)
  - **68 – 95 – 99.7 rule**
  - \( z \)-scores, using the table
  - Simpson’s paradox (batting averages, alleged discrimination case)
  - sources of bias (opportunity sample, response bias, sampling bias, etc.)
- yes/no surveys
  * $p$ and $\hat{p}$, mean and standard deviation of $\hat{p}$
  * margin-of-error, confidence intervals

- Decision theory
  - **Drawing and solving decision trees** (branches, decision nodes, chance nodes)
  - legal, medical, and business examples
  - risk aversion, certainty equivalent, *Deal or No Deal*

- Voting theory
  - **Voting systems for multi-candidate elections**
    * Plurality voting
    * Approval voting
    * Ranked voting
      - instant run-off
      - Borda count
    * range (score) voting
  - Condorcet criterion
  - Clone independence of a voting system
  - Voting paradoxes; independence on irrelevant alternatives
  - manipulable voting systems; strategic vs. sincere voting
  - **Arrow’s impossibility theorem**
    * applications to organ donation procedures
  - Weighted voting
    * dictators, veto power, dummy voters
    * Shapley–Shubik power index
    * applications to the electoral college
    * understanding Google’s PageRank algorithm via weighted voting

- Game theory
  - Basics: players, games, strategies, payouts, information
  - dominant, dominated strategies
  - pure strategies, **mixed strategies**
  - **Nash equilibrium**
  - $2 \times 2$ zero sum games
    * maximin, minimax, checking for saddle points
    * optimal mixed strategies
    * value of game
* sports examples: baseball, soccer
  - $m \times n$ zero sum games
    * elimination of dominated strategies
    * maximin, minimax, checking for saddle points
    * weighted rock-paper-scissors
    * Blotto game
    * Pirates and gold coins
  - non-zero sum games
    * prisoner’s dilemma; real-life examples
    * chicken
    * stag hunt

• Auctions
  - English auctions, Dutch auctions, first-price sealed-bid
  - Vickrey auctions (second-price sealed-bid), dominant strategy for bidder
  - all-pay auctions
  - winner’s curse

• Fair division
  - cake cutting, triangular cakes, splitting the rent
  - adjusted winner procedure
  - Knaster inheritance procedure