1. [#14.24] Let \( f(x) := x^2 - 4x + 6 \) and let \( x_n \) be a sequence defined by the recurrence
\[ x_{n+1} = f(x_n), \]
that is, \( x_{n+1} = x_n^2 - 4x_n + 6 \).

a) If \( \lim_{n \to \infty} x_n \) exists and equals \( L \), what possible values can \( L \) have?
b) The behavior as \( n \to \infty \) depends on the initial value, \( x_0 \). For each \( x_0 \in \mathbb{R} \), describe this behavior. [Hint: Graph the functions \( y = x \) and \( y = f(x) \) and interpret the graphs. In this example, it may help to rewrite \( y = f(x) \) as \( y - 2 = (x - 2)^2 \).]

2. [#14.27] For \( c > 0 \), let \( x_n = \left(\frac{c^n + 1}{n}\right)^{1/n} \). Determine \( \lim x_n \). More generally find \( \lim_{n \to \infty} (a^n + b^n)^{1/n} \). [Hint: First consider the case \( c < 1 \) and use the Squeeze Theorem.]

3. [#14.30] Let \( x_n \) be the sequence defined recursively by \( x_1 = 1 \) and \( x_{n+1} = \frac{1}{x_1 + \cdots + x_n} \). Prove that this sequence converges and obtain the limit.

4. [#14.33] If \( \sum_{k=1}^{\infty} a_k \) converges to \( A \) and \( \sum_{k=1}^{\infty} b_k \) converges to \( B \), show that \( \sum_{k=1}^{\infty} (a_k + b_k) \) converges to \( A + B \).

5. [#14.36] Find the rational number whose repeating decimal expansion is \( .247247247\ldots \). Also, find the rational number whose repeating octal (that is, base 8) expansion is \( .247247247\ldots \).

6. [#14.43] Compute \( \sum_{k=1}^{\infty} \left(\frac{x}{x+1}\right)^k \). What assumptions must be made about \( x \)?

7. [Telescoping Series #14.44]

a) Compute \( \sum_{n=1}^{N} \frac{1}{n(n+1)} \) and then \( \sum_{n=1}^{\infty} \frac{1}{n(n+1)} \).

Hint: Use the partial fraction decomposition \( \frac{1}{(x-a)(x-b)} = \frac{1}{a-b} \left( \frac{1}{x-a} - \frac{1}{x-b} \right) \).

b) Use this to estimate \( \sum_{n=1}^{\infty} \frac{1}{n^2} \)

8. [#14.45] Let \( S_n := a_1 + \cdots + a_n \). If \( S_n = 1/n \) for all \( n \geq 1 \), find \( a_n \) for all \( n \geq 1 \).

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