Big Numbers

• Age of the universe (14 billion years) in seconds:

\[ N = 14 \times 10^9 \text{ years} \times \frac{365 \text{ days}}{\text{year}} \times \frac{24 \text{ hours}}{\text{day}} \times \frac{3600 \text{ seconds}}{\text{hour}} = 441,504,000 \times 10^9 \approx 4.4 \times 10^{17} \text{ seconds}. \]

• 1 billion = 1,000,000,000 = 10^9

• \(2^{121} = (2^{11})^{11} = 2048^{11} \approx 2.7 \times 10^{36}\)

• \(9^{55} = (9^5)^{11} = 32805^{11} \approx 3.0 \times 10^{52}\)

• \(7^{88} = (7^8)^{11} = 5764801^{11} \approx 2.3 \times 10^{74}\)

• \(e^{100} \approx 2.68 \times 10^{43}\)

• \(15! \approx 1.3 \times 10^{12}\) \quad 25! \approx 1.5 \times 10^{25}\)

Example 1. Let \(S_N := 1 + \frac{1}{2} + \cdots + \frac{1}{N}\). Find an estimate for \(N\) so that \(S_N > 100\).

**Answer** By the idea behind the **integral test**

\[ \ln(N+1) < S_N < 1 + \ln N \]

Thus, to insure that \(S_N > 100\) pick \(\ln(N+1) > 100\), that is, \(N+1 > e^{100} \approx 2.7 \times 10^{43}\).

On the other hand, if \(1 + \ln N < 100\), then \(S_N < 100\). Here we can pick any \(N < e^{99} \approx 10^{43}\).

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1 R. P. Boas, Jr. and J. W. Wrench, Jr. found the exact number:

\[1509268862211378323693563264538101449859497\]

Example 2. How many multiplications are needed to compute the determinant of a $25 \times 25$ matrix of real numbers?

Answer: $25! \approx 10^{25}$

Example 3. Consider a stock that is sold on the New York Stock Exchange. If someone is Paris places a buy order and $1/100$ of a second later someone in New York buys the same stock, which order is executed first?

Some Data:
Speed of light in a vacuum $\approx 300,000$ km/sec
$\approx 186,000$ miles/sec
$\approx 1$ foot/ns $\approx 1$ m/$3.3$ ns (ns = nanoseconds)
$\approx 4,000$ miles/.0215 sec (3,625 miles from Paris to New York)
**Significant Digits**

Say we have the data \( a := 12.47 \) and \( b := 7.2 \), both rounded off. Thus the absolute value of the error in \( a \) is at most \( e_a : 0.005 \) while \( e_b := 0.05 \). Now \( ab = 89.784 \). How trustworthy is this? Since

\[
12.465 < a < 12.475 \quad \text{while} \quad 7.15 < b < 7.25,
\]

we know that

\[
89.12475 < ab < 90.44375.
\]

We should discard most of the digits and use the rounded number

\[
ab \approx 89.8.
\]

To better understand the error, note that

\[
(a + e_a)(b + e_b) = ab + ae_b + be_a + e_ae_b \approx ab + ae_b + be_a.
\]

The larger number, \( a \), happens to be multiplied by the larger error, \( e_b \). Although \( a \) has 4 significant digits, \( b \) only has 2. The analysis changes considerably if you are given that \( b = 7.20 \) since then \( e_b = 0.005 \).