Problem Set 7

DUE: Thurs. March 19 in class. [Late papers will be accepted until 1:00 PM Friday.]
This week. Please read Chapter 7, Sections 7.1 – 7.7 in the Haberman text.

1. p. 162 #5.3.3 [First try the special case \( \alpha(x) = 2x \).]

2. p. 163 #5.3.8 (& p. 188 #5.6.2)

3. p. 163 #5.3.9

4. p. 166 #5.4.1

5. p. 174 #5.5.1(a-f)

6. p. 175 #5.5.2

7. p. 188 #5.6.1(a)

8. Let \( A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \). Note that \( A \) is symmetric.
   a) Find the eigenvalues \( \lambda_j \), \( j = 1, 2, 3 \) and corresponding orthogonal eigenvectors \( \vec{v}_j \).
   b) Use this to find the general solution of the homogeneous equation

\[
\frac{d\vec{x}}{dt} = A\vec{x}.
\]

SUGGESTION: Seek the solution as a linear combination of the eigenvectors

\[
\vec{x}(t) = \phi_1(t)\vec{v}_1 + \phi_2(t)\vec{v}_2 + \phi_3(t)\vec{v}_3,
\]

where the coefficients \( \phi_j(t) \) are found as follows:
(i). Use equation (2) to compute both \( \frac{d\vec{x}}{dt} \) and \( A\vec{x} \). Then, for equation (1) equate these to deduce that

\[
\frac{d\phi_1(t)}{dt} = \lambda_1\phi_1(t), \quad \frac{d\phi_2(t)}{dt} =?, \quad \frac{d\phi_3(t)}{dt} =?
\]

(ii). Now solve these equations for \( \phi_1(t) \), \( \phi_2(t) \), and \( \phi_3(t) \). Then equation (2) gives the desired solution.
c) Find the solution of (II) that also satisfies the initial condition $\vec{x}(0) = (1, 0, 0)$

9. [Ignore. It repeated #7 above]

10. p. 272 #7.2.2 [correction of typo 2.7.2]

11. p. 279 #7.3.4(a)

[Last revised: March 16, 2015]