Problem Set 9

Due: Thurs. April 9 in class. [Late papers will be accepted until 1:00 PM Friday.]

This week: Please read Chapter 8 in the Haberman text.

1. [See Section 7.3 in Haberman] Solve the wave equation $u_{tt} = c^2 \Delta u$ in the square \( \Omega = \{0 < x < \pi, \ 0 < y < \pi\} \) in the plane with \( \nabla u \cdot N = 0 \) on the boundary and initial conditions $u(x, y, 0) = 0$, $u_t(x, y, 0) = \cos 2x \cos 5y$.

2. [See Section 7.3 in Haberman] In the square $0 \leq x \leq a$, $0 \leq y \leq a$ in the plane, a substance is diffusing whose molecules multiply at a rate proportional to the concentration. It thus satisfies

   \[ u_t = k \Delta u + \gamma u, \]

   where $k$ and $\gamma$ are constants. Assume that $u = 0$ on all four sides of the square. What is the condition on $\gamma$ so that the concentration does not grow without bound?

3. p. 287 #7.4.1

4. p. 290 #7.5.2

5. p. 291 #7.5.6 [This generalized Eq. 7.5.7 on page 289]

6. p. 338 #7.10.1(a)

7. p. 339 #7.10.2(b)

[Last revised: April 10, 2015]