This example is similar to Problem Set 7 # 3a).

**Example:** Find a function \( u(x, y) \) that satisfies \( u_x + 3u_y = 0 \) with \( u(0, y) = 1 + e^{2y} \).

**Solution:** The differential equation can be written \( \nabla u \cdot V = 0 \) where \( V = (1, 3) \). It means that at every point the directional derivative in the direction of \( V \) is 0 so \( u(x, y) \) is constant along these parallel straight lines, which have the form \( y = 3x + C \). Given a point \((x, y)\) one computes \( y - 3x \) to determine \( C \), that is, which line you are on.

Thus the solution \( u(x, y) = h(y - 3x) \) for some as yet unknown function \( h(s) \). Now we use the initial condition \( u(0, y) = 1 + e^{2y} \). It gives us

\[
1 + e^{2y} = u(0, y) = h(y).
\]

Consequently \( u(x, y) = 1 + e^{2(y-3x)} \).

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