Math 312, Homework 3 (due Friday, Sep. 28)

Name:________________________________ (if you choose to use this as a coversheet)

Reading

- Read sections chapter 3.1–3.3 of Bretscher.

Book problems NOTE: you can ignore the "GOAL" paragraph at the start of each exercise set in Bretscher – this is not a set of instructions for particular problems.

- Section 3.1, problems 12, 20, 22, 34, 36, 39, 40
- Section 3.2, problems 2, 6, 8, 17, 18, 34, 40, 54
- Section 3.3, problems 16, 22, 28, 80

Additonal problems

1. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ and $S : \mathbb{R}^2 \to \mathbb{R}^3$ be linear tranformations, so $S \circ T : \mathbb{R}^3 \to \mathbb{R}^3$ and $T \circ S : \mathbb{R}^2 \to \mathbb{R}^2$. Let the matrix of T be B and let the matrix of S be A.
   (a) Why must there be a vector $\vec{x} \in \mathbb{R}^3$ such that $B\vec{x} = 0$?
   (b) Prove that AB (a 3×3 matrix) can never be invertible.
   (c) Give an example to show BA (a 2×2 matrix) may be invertible.
2. Are the following subsets of $\mathbb{R}^2$ actually subspaces? Explain.
   (a) $\{(x, y) \mid xy = 0\}$
   (b) $\{(x, y) \mid x \text{ and } y \text{ are integers}\}$
   (c) $\{(x, y) \mid x + y = 0\}$
   (d) $\{(x, y) \mid x + y \geq 0\}$
3. Let T be a linear transformation with trivial kernel. Prove that if $T(\vec{x}) = T(\vec{y})$, then $\vec{x} = \vec{y}$. (Hint: use subtraction.) Such transformations T are called *one-to-one*.
4. Prove that if vectors $\vec{v}_1, \dots, \vec{v}_n$ are linearly dependent, then one of these vectors is a linear combination of the others.
5. Prove that the span of vectors $\vec{v}_1, \dots, \vec{v}_k$ in $\mathbb{R}^m$ is always a subspace of $\mathbb{R}^m$.
6. Say $\vec{v}_1, \dots \vec{v}_k$ are linearly independent in $\mathbb{R}^n$. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be linear. Show by example that $T(\vec{v}_1), \dots, T(\vec{v}_k)$ need not be linearly independent. However, prove that these vectors are linearly independent if $\ker(T)$ is trivial.
7. Use Theorems from section 3.3 (or from class) to explain carefully why if V and W are subspaces with V contained inside of W, then $\dim V \leq \dim W$.