Math 312, Homework 9 (due Monday, November 19)

Name:________________________________ (if you choose to use this as a coversheet)

Reading Section 7.6 and section 5.1 – 5.4 of Bretscher.

Book problems

- Section 7.6, problems 4, 10, 25, 26
- Section 5.1, problems 15, 16, 17, 19ab, 26, 28
- Section 5.2, problems 2, 6, 14, 32, 33
- Section 5.3, problems 5, 6, 7, 8, 10, 33, 36, 37
- Section 5.4, problems 21, 22

Additional Problems

1. Let $\mathbb{P}_n(\mathbb{C})$ denote the set polynomials with complex coefficients. This is a complex vector space, and also a real vector space. Give a basis for $\mathbb{P}_2(\mathbb{C})$ as a) a real vector space and b) a complex vector space.
2. Let A be a 2×2 matrix whose eigenvalues are not real.
   (a) Suppose one of the eigenvalues has modulus 1. Explain why the other must as well.
   (b) Explain why A must be diagonalizable.
3. Let W be a subspace of $\mathbb{R}^n$. Prove that $W^\perp$ is also a subspace of $\mathbb{R}^n$.
4. Prove that $W = (W^\perp)^\perp$. (Hint: first prove that spaces have the same dimension. Second, prove that W is contained in $(W^\perp)^\perp$.)
5. Let A be a skew-symmetric matrix (meaning $A^T = -A$). Prove that $A\vec{x}$ is always orthogonal to $\vec{x}$. (For instance, this includes the case where A is rotation by $90°$.) (Hint: use the fact that $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$.)