Question 1: {15 pts}

Consider the following system of linear equations:

\[
\begin{align*}
3w + x + 2y + z &= -4 \\
w + 4x + y + 3z &= 13 \\
5w - 2x + 3y - z &= -21
\end{align*}
\]

Give the complete solution to this system – show your work.

Question 2: {15 pts}

If A is a square matrix such that \((I - A)\) is nonsingular prove that:

\[
A(I - A)^{-1} = (I - A)^{-1}A
\]

Question 3: {15 pts}

Suppose two matrices, A and B, are row equivalent, that is there exists a nonsingular matrix P such that PA = B. Answer the following questions about A and B. Explain your answer in each case.

- Do A and B have the same column space ie does: \(R(A) = R(B)\)
- Do A and B have the same null space ie does: \(N(A) = N(B)\)
- Do A and B have the same row space ie does: \(R(A^T) = R(B^T)\)
- Do A and B have the same left hand null space ie does: \(N(A^T) = N(B^T)\)

Question 4: {15 pts}
Show that the following set of vectors constitute a basis for $\mathbb{R}^3$.
\[
\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix}
\]
Compute the coordinates of the following vectors with respect to this basis:
\[
\begin{pmatrix} 3 \\ 5 \\ -5 \end{pmatrix}, \begin{pmatrix} 6 \\ 2 \\ -2 \end{pmatrix}
\]

Question 5: \{15 pts\}
If $x$ and $y$ are vectors such that $\|x - y\|_2 = \|x + y\|_2$, what is $x^Ty$?

Question 6: \{15 pts\}
If $A$ is a square matrix prove that
\[
\|A\|_F^2 = \|A^+\|_F^2 + \|A^-\|_F^2
\]
where $\|A\|_F^2 = tr(A^TA)$ denotes the square of the Frobenius norm of $A$, $A^+ = \frac{A + A^T}{2}$ denotes the symmetric part of $A$ and
\[
A^- = \frac{A - A^T}{2}
\]
denotes the skew symmetric part of $A$