Problem Set 2

Due: Thurs. Sep. 13 in class. [Late papers accepted (without penalty) until 1:00 PM Friday.]

Please read Sections 2.1 – 2.5 of Burden-Faires

Below are lots of Exercises (all from the 9th Edition of Burden-Faires). Fortunately most of them are short.

Problems

1. [B-F p. 39 #8] This concerns computing \( \sum_{k=1}^{n} \sum_{j=1}^{k} a_k b_j. \)
   
   a) How many multiplications and additions are needed?
   
   b) Modify the above sum to an equivalent form that reduced the number of computations.

2. [Sec. 2.1 #1] Use the bisection method to find \( p_3 \) for \( f(x) = \sqrt{x} - \cos x \) on \([0, 1]\).

3. [Sec. 2.1 #10] Let \( f(x) := (x + 2)(x + 1)x(x - 1)(x - 2) \). To which zero of \( f \) does the Bisection method converge when applied to the following intervals?
   
   a) \([-1.5, 2.5]\]  
   b) \([-0.5, 3.4]\]  
   c) \([-0.5, 3]\]  
   d) \([-3, -0.5]\]

4. [Sec. 2.1 #15] Use Theorem 2.1 to find a bound for the number of iterations needed to achieve an approximation with accuracy \( 10^{-4} \) to the solution of \( x^3 + x - 4 = 0 \) lying in the interval \([1, 2]\). Find an approximation to the root with this degree of accuracy.

5. [Sec 2.2 #5] Use a fixed point iteration method to determine a solution accurate to within \( 10^{-2} \) of \( x^4 - 3x^2 - 3 = 0 \) on \([1, 2]\). Use \( p_0 = 1 \).

6. [Sec 2.2 #8] Use Theorem 2.3 to show that \( g(x) = 2^{-x} \) has a unique fixed point on \([\frac{1}{2}, 1]\). Use fixed point iteration to find an approximation to the fixed point accurate to within \( 10^{-4} \). Use Corollary 2.5 to estimate the number of iterations required to achieve \( 10^{-4} \) accuracy, and compare this theoretical estimate to the number actually needed.

7. [Sec 2.2 #20a] If \( A \) is any positive number, show that the sequence defined by
   
   \[ x_n = \frac{1}{2} x_{n-1} + \frac{A}{2x_{n-1}}, \quad \text{for} \quad n = 1, 2, \ldots \]
   
   whenever \( x_0 > 0 \).
8. [Sec 2.2 #24] Let \( g \in C^1[a, b] \) and \( p \) be in \((a, b)\) with \( g(p) = p \) and \(|g'(p)| > 1\). Show that there exists a \( \delta > 0 \) such that if \( 0 < |p_0 - p| < \delta \), then \( |p_0 - p| < |p_1 - p| \). Thus, no matter how close the initial approximation \( p_0 \) is to \( p \), the next iterate, \( p_1 \), is further away so the fixed point iteration does not converge if \( p_0 \neq p \). In this case \( p \) is called a repulsive fixed point.

9. [Sec. 2.3 #6a) Use Newton’s method to find a solution accurate to within \(10^{-5}\) of

\[
e^x + 2^{-x} + 2 \cos x - 6 = 0 \quad \text{for} \quad 1 \leq x \leq 2.
\]

10. [Sec. 2.3 #20(d,e,f)] The equation \( x^2 - 10 \cos x = 0 \) has two solutions \( x_1 \) and \( x_2 \). Use Newton’s method to approximate them to within \(10^{-5}\) with the following values of \( p_0 \):

d). \( p_0 = 25 \), e). \( p_0 = 50 \), f). \( p_0 = 100 \).

11. [Sec. 2.3 #24. This is the example at the beginning of Chapter 2, p. 47-48]. Find an approximation for \( \lambda \), accurate to within \(10^{-4}\) fr the population equation

\[
1,564,000 = 1,000,000e^\lambda + \frac{435,000}{\lambda}(e^\lambda - 1)
\]

Use this value to predict the population at the end of the second year – assuming that the immigrate ion rate this year remains at 435,000 people per year.

12. [Sec. 2.3 #31] The logistic population growth model is described by an equation of the form

\[
P(t) = \frac{P_L}{1 - ce^{kt}},
\]

where \( P_L, c, \) and \( k \) are positive constants. Here \( P(t) \) is the population at time \( t \), \( P_L = \lim_{t \to \infty} \) is the limiting value of the population,

a) Use the census data for 1950, 1960, and 1970 [see the table on page 105) to determine the constants \( P_L, c \) and \( k \).

b) Then use the equation (with \( t = 0 \) in 1950) to predict the population for 1980 and 2010. Compare the 1980 prediction with the actual value.

13. Compute \( \sqrt{3} \) by the following methods:

a) bisection [Sec. 2.1 #12]

b) iteration [Sec. 2.2 #9]

c) Newton’s method [Sec. 2.3]

d) Write a few sentences comparing your results.

[Last revised: September 8, 2018]