Problem Set 12

DUE: Thurs. Apr. 23 in class. [Late papers will be accepted until 1:00 PM Friday.]

This week: Please read Chapter 12.3 and reread Chapter 11 in the Strauss text.

1. Strauss p. 277 #2

2. Strauss p; 278 #10

3. Strauss p; 281 #1

4. Let Ω in $\mathbb{R}^2$ be a bounded region and let $\hat{\Omega} \subset \mathbb{R}^2$ be the region obtained by stretching the x and y coordinates by a factor $c > 0$. Thus $\hat{x} = cx$ and $\hat{y} = cy$. The λ_n and v_n be the eigenvalues and corresponding eigenfunctions of Ω.
   a) What can you say about the eigenvalues and eigenfunctions of $\hat{\Omega}$?
   b) Repeat the analogous problem for a region Ω in $\mathbb{R}^3$?

5. Strauss p. 304 #1

6. Strauss p; 304 #4

7. Let $\Omega \subset \mathbb{R}^2$ be a region inside the rectangle with vertices at $(-1,-1)$, $(2,-1)$, $(2,2)$, and $(-1,2)$, and assume the square with vertices at $(0,0)$, $(1,0)$, $(1,1)$, and $(0,1)$ is inside Ω. Use this information to estimate the lowest eigenvalue of the Laplacian for the region Ω with boundary values zero. Thus, find numbers $0 < m < M$ so that
$$m < \lambda_1(\Omega) < M.$$

8. Strauss p. 309 #1

9. Strauss p. 309 #9

10. Strauss p. 313 #2

11. Strauss p. 313 #3

[Last revised: April 17, 2015]