Part A: Examples (4 problems, 5 points each so 20 points). Give an example having the specified property.

1. A subset of the real line $\mathbb{R}$ that contains all but three of its limit points.

2. A sequence of real numbers $x_n$ with the property that $|x_{n+1} - x_n| \to 0$ but the sequence does not converge.

3. A collection of closed intervals $A_k \subset \mathbb{R}$, $k = 1, 2, \ldots$, whose union is not closed.

4. A collection of bounded nested intervals $J_1 \supseteq J_2 \supseteq J_3 \supseteq \cdots$ whose intersection is empty.

Part B: Short Problems (4 problems, 9 points each so 36 points)

B–1. Let $a_k = \sqrt{k+1} - \sqrt{k}$, $k = 1, 2, \ldots$. Determine if this sequence converges.

B–2. Show that $1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ldots + \frac{1}{k!} + \ldots < 2$.

B–3. Given any two rational numbers $p < q$, prove there is an irrational number $c$ between them.

B–4. Given any $c > 1$, show that $\frac{c^n}{n!} \to 0$.

Part C: Traditional Problems (3 problems, 15 points each so 45 points)

C–1. Let $x_k \geq 0$ be a sequence of real numbers that converges to $A$. Show that $A \geq 0$.

C–2. Given a sequence $\{a_k\}$ of real numbers, let $S_n = \frac{a_1 + \cdots + a_n}{n}$ be the sequence of averages (arithmetic mean). If $a_k$ converges to $0$, show that the averages $S_n$ also converge to $0$. 
C–3. Let \( \{a_n\} \in \mathbb{C} \) be a \emph{contracting} sequence, that is there is a \( 0 < c < 1 \) so that
\[
|a_{n+1} - a_n| \leq c|a_n - a_{n-1}|, \quad n = 1, 2, 3, \ldots
\]

\begin{enumerate}
\item[a)] Show that \( |a_{n+1} - a_n| \leq c^n|a_1 - a_0| \).
\item[b)] If \( n > k \), show that \( |a_n - a_k| \leq \frac{c^k}{1 - c}|a_1 - a_0| \).
\item[c)] Show that the sequence \( a_n \) converges.