Relative Topology and Connectedness

Let $S \subset \mathbb{R}^n$. and $p \in S$.

A **neighborhood of $p$ relative to $S$** is a set $T \subset S$ that contains $B(p, r) \cap S$ for some $r > 0$.

$V \subset S$ is **open** relative to $S$ if $V = S \cap U$ where $U$ is an open set in $\mathbb{R}^n$.

The following are equivalent:

- $F$ is closed relative to $S$.
- $F = S \cap K$, where $K$ is closed in $\mathbb{R}^n$
- If $x_j$ is a sequence of points in $F$ that converge to a point $x \in S$, then $x \in F$.
- The complement of $F$ relative to $S$, $S - F$, is open relative to $S$.

A set $S$ is **disconnected** if there are two **disjoint** non-empty sets $S_1$ and $S_2$ such that $S = S_1 \cup S_2$ and both $S_1$ and $S_2$ are closed relative to $S$. 
The only connected sets in $\mathbb{R}$ are intervals (possibly infinite):

$$a < x < b, \quad a \leq x \leq b, \quad a \leq x < b, \quad a < x \leq b$$

A function $f$ is **continuous at a point** $p \in S$ if for every neighborhood $V$ of $f(p)$, then the inverse image $f^{-1}(V)$ is a neighborhood of $p$ relative to $S$.

In brief, if the inverse image of every open set is open.