

MATH 240 QUIZ 10

Name: _____

Question:

Solve the following initial-value problem:

$$y'' - 2y' + 2y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

Extra credit(+1): Does the above equation describe motion of mass on a spring? If so, state whether the motion is underdamped, critically damped, or overdamped.

Solution:

The auxiliary polynomial $r^2 - 2r + 2 = 0$ has root $1 \pm i$, therefore the general solution is

$$y(t) = c_1 e^t \cos t + c_2 e^t \sin t$$

plug in the initial conditions, we get

$$c_1 = 1, c_2 = 1$$

so

$$y(t) = e^t \cos t + e^t \sin t$$

This does not describe motion of mass on a spring, because in the equation, y' term has negative coefficient, while the equation of spring-mass system should have non-negative coefficients.

One can also see this from solution, as $y(t)$ can approach to infinity when $t \rightarrow +\infty$, and it violates conservation of energy (this system has no external force, so total energy is conserved).