Problem 1 (10 points)
Find the area of the surface of revolution obtained by rotating \( y = \sqrt{4x - x^2 - 3} \)
(a) around the \( x \)-axis,
(b) around the \( y \)-axis.

Problem 2 (10 points)
Find the length of the curve: \( x = \sqrt{1 - y^2}, \ 0 \leq y \leq 1 \).

Problem 3 (10 points)
Describe the curve given in Cartesian coordinates:
\[ x^2 + 2y^2 = 1, \]
by an equation in polar coordinates \((r, \theta)\). Check your polar equation for \( \theta = 0, \ \theta = \frac{\pi}{4}, \ )\text{ and } \( \theta = \frac{\pi}{2} \).

Problem 4 (10 points)
Consider the following power series:
\[ 1 - x + x^2 - x^3 + x^4 - x^5 + x^6 - x^7 + \ldots \] (1)
(a) Find the radius of convergence \( R \).
(b) Does the series converge or diverge for \( x = -R \), and likewise for \( x = R \) ?
(c) Find a simple expression for the sum of the series when \( |x| < R \).

Problem 5 (13 points)
Consider the following power series:
\[ 1 - 2x + 3x^2 - 4x^3 + 5x^4 - 6x^5 + 7x^6 - 8x^7 + \ldots \] (2)
(a) Find the radius of convergence \( R \).
(b) Does the series converge or diverge for \( x = -R \), and likewise for \( x = R \) ?
(c) Find a simple expression for the sum of the series when \( |x| < R \).
Problem 6 (11 points)

(i) Find the Maclaurin series \( \sum_{n=0}^{\infty} a_n x^n \) of the function: \( \sin^2 x \).

(ii) Find the Maclaurin series \( \sum_{n=0}^{\infty} b_n x^n \) of the function: \( \cos^2 x \).

(iii) Check that: \( \sum_{n=0}^{\infty} a_n x^n + \sum_{n=0}^{\infty} b_n x^n = 1 \).

Problem 7 (11 points)

How many terms of the Maclaurin series of \( \sin^2 x \) must you use in order to calculate \( \sin^2 \frac{\pi}{4} \) with an error less than 0.01?

Problem 8 (10 points)

Find the Maclaurin series of the following functions: \( (1+x)^3 \), \( (1+x)^4 \), \( (1+x)^5 \), and \( \sqrt{1+x} \).

Problem 9 (15 points)

(a) Is it true or false\(^1\) that the following sequence \( x_1, x_2, \ldots, x_n, \ldots \) is increasing:

\[
x_n := 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \ln(n+1).
\]

(b) Is it true or false\(^1\) that for any \( n \geq 2 \):

\[
\frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} < \ln n.
\]

(c) Prove the existence of the following limit:

\[
\lim_{n \to \infty} (1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \ln n)
\]

\(^1\)Explain your answer