Ideas in Mathematics
Math 170, Spring 2016
Assignment 2, part 1

1. The numbers below are written in binary; convert them to decimal notation.

- 10100010 ______ 162
- 00000000 ______
- 11001110 ______
- 00110100 ______
- 00100001 ______

The numbers below are written in decimal; convert them to binary notation.

- 17 ______ 10001
- 25 ______
- 31 ______
- 40 ______
- 63 ______

2. Write each set below using set-builder notation; there is more than one correct answer.

- $\{0, 1, 2, 3, \ldots\}$ ______ $\{n - 1 : n \in \mathbb{N}\}$
- $\{\ldots, -3, 0, 3, 6, \ldots\}$ ______
- $\{1, 1/2, 1/3, 1/4, \ldots\}$ ______
- $\{1, 2, 4, 8, 16, \ldots\}$ ______
- $\{7, 8, 9, \ldots\}$ ______

List several elements and an ellipses to provide an alternate description of these sets.

- $\{n^2 : n \in \mathbb{N}\}$ ______ $\{1, 4, 9, 16, \ldots\}$
- $\{5n + 1 : n \in \mathbb{N}\}$ ______
- $\{x \in \mathbb{Z} : -2 < x\}$ ______
- $\{x \in \mathbb{Q} : x^2 = 9\}$ ______
- $\{n^2 = 2 : n \in \mathbb{N}\}$ ______

3. The **cardinality** $|A|$ of a finite set A is its number of elements. Determine each of the indicated cardinalities; all sets are finite. Some of these questions require careful thinking; a correct answer is provided for the first problem.

- $|\{1, 2, 3, 4, 5\}|$ ______ 5
- $|\{\}|$ ______
- $|\{\text{red}, \text{yellow}\} \cup \{\text{blue}\}|$ ______
- $|\{\emptyset\}|$ ______
- $|\{1, \{2, 3\}, \{1, 2, \{3\}\}\}|$ ______
- $|\{\emptyset, \{\{1\}\}\}|$ ______
- $|\emptyset \cap \{2, 3\}|$ ______
- $|\{1, 2, 2, 3, 5, 5\}|$ ______
- $|\{5, 1\} \cup \{1, 2, 5\}|$ ______
- $|\{\mathbb{N}, \mathbb{Z}, \mathbb{Q}\}|$ ______

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Assignment 2, part 2

The following questions will require more thinking than those in part 1. Even in math, not every question has a good answer. You might think and think and think and still not figure anything out. In cases where you cannot come up with a good answer, spend a few sentences explaining what you tried and why that didn't work out as you had hoped.

As indicated in the syllabus, working with other people is highly encouraged. That said, the final answers you write down should be your own. Typed answers are preferred.

1. In the decimal system, digits to the right of a decimal indicate the number of 1's, 10's, 100's, etc, while digits to the left of it indicate the number of tenths, hundredths, thousandths, etc. Notice that these are all powers of tens: $\ldots, 10^{-3}, 10^{-2}, 10^{-1}, 10^{0}, 10^{1}, 10^{2}, 10^{3}, \ldots$. The same convention is followed for other bases.

   (a) Show how to convert the following four numbers from base 5 into standard decimal (base-10) notation: 1.111, 1.234, 3.141, 0.4321.

   (b) Show how to convert the following four numbers from standard decimal (base-10) into base 5 notation: 1.2, 4.4, 5.5, 3.14.

2. Consider the additive Roman numeral system with seven symbols I, V, X, L, C, D, and M. If you double a number, you often need twice as many digits to represent it. For example, two times II is IIII, and two times XI is XXII; two times MMMV is MMMMMMX, which uses almost twice as many digits. Sometimes however, doubling a number can decrease the number of digits needed, such as in doubling XXV to L.

   (a) In the decimal system, can doubling a number double the number of digits necessary to represent it? If yes, provide examples; if no, explain why not.

   (b) In the decimal system, can doubling a number decrease the number of digits necessary to represent it? If yes, provide examples; if no, explain why not.

3. Let $A = \{25a + 13b : a, b \in \mathbb{Z}\}$ and $B = \mathbb{Z}$. Are A and B the same? If they are, show that they are the same; if they are not, provide an example of an element of A that does not belong to B, or else an element of B that does not belong to A.

   Read Chapter 2 "Patterns of the Mind" from *The Language of Mathematics: Making the Invisible Visible*, by Keith Devlin.